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Abstract

In 23 out of 26 U.S. industries, the annual CEO bonus is larger than the annual salary, suggesting that the bonus strongly affects the CEO's decisions. As the high leverage of financial institutions is often blamed for the 2008 financial crises, in this study we focus on leverage as a factor determining risk, particularly in finnacial institutuions. The typical bonus scheme is not a leverage-neutral bonus scheme (LNBS), as the agent's optimal policy is to employ a corner solution: either zero or exteremely high leverage. Thus, consistent with Ross (2004), the bonus scheme does not neccesarily induce the agent to take greater risks. However, although more leverage is not preffered by all preferences, in most cases it is preffered. Thus, we suggest a combination of incentive parameters, which makes the agent indifferent to leverage, thereby preventing conflict beween the agent and the principal (stockholders).

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Introduction

Much ink has been spilled over the current crisis in the financial markets. Economic cycles probably cannot be avoided and will most likely reoccur in the future. However, greediness and, correspondingly, the taking of relatively large risks by CEOs, have been cited as two of the main reasons for the intensity of the current crisis (see Bebchuk and Spamann, 2010). As greediness of the CEOs and excessive leverage are usually mentioned in one breath, this assertion implicitly implies that, according to the existing compensation scheme, it is beneficial for CEOs to take excessive leverage (see, e.g., Dong, Wang, and Xie, 2010) .

Not all researchers, however, agree with these conclusions. Ross (2004) mathematically proves that the existing bonus schemes, particularly the option-like convex incentive scheme, do not necessarily induce all risk-averse agents, independent of their specific preferences, to take greater risks (see also Carpenter, 2000). Yet, the common folklore asserting that the typical incentive schemes induce agents to take greater risks really does exists and is definitely alive and kicking. If indeed in practice most agents, albeit not all agents, take greater risks due to the existing incentive schemes, it implies that most CEOs who have incentive schemes take excessive risks, which serves as a base for the assertion regarding the relation between incentives and risk-taking.[1]

In an attempt to prevent CEOs from taking excessive risks in the future, one of the components of the suggested rescue plan in the U.S., as well as in other

[1] The empirical evidence strongly supports the approach that managerial incentives have observable operational and policy implications. Coles, Naveen and Lalitha (2005), for example, find a strong causal relation between managerial compensation and strategic policies, including leverage policy. This approach has lead John, Sounders and Senbet (2000) to conclude that the compensation structure should be an input in bank pricing of deposit insurance.

countries, relates to the CEO's compensation scheme.[2] President Obama, for example, suggests imposing a cap on the compensation paid to CEOs. Similarly, U.S. Treasurer Secretary, Tim Geithner, suggests imposing changes in the compensation scheme, emphasizing long-term incentives, rather than short-term incentives in an attempt to "*prevent excessive risk taking*".[3] As the bonus is the main short-term incentive, which in most cases is even larger than the annual salary (see Table 1 below), the U.S. Treasurer Secretary indirectly refers to changes that need to be implemented in the bonus scheme.

The typical CEO compensation package contains the following main components: annual salary, annual bonus, other benefits, and stocks gains. Table 1 illustrates the relative share and, correspondingly, the relative importance of each of the above components in an average CEO's total compensation package.

<< Insert Table 1 >>

As can be seen from this table, the bonus component, which varies across industries, is relatively large. Actually, in 23 out of the 26 U.S. industries in 2007, the median bonus was larger than the annual salary. Yet, the economic literature has mainly focused on other components, while paying little attention to the bonus component. In a recent study, for example, Benmelech, Kandel and Veronesi (forthcoming) show that with asymmetric information, stock-based compensation induces managers to exert costly effort, and to conceal bad news about future growth options, and choose sub-optimal investment policies to support the pretense. As regards to stock options, Pukthuanthong, Roll, and Walker (2007), for example, find that companies perform better when managers receive a balanced combination of stock option grants and

[2] Many of the suggested provisions regarding management compensation have already been imposed on firms which have benefited from the Government TARP money.
[3] From an interview on Bloomberg TV's "Political Capital with Al Hunt" (see also, *The Note*, May 22 by ABC News reporters Matthew Jaffe and Rick Klein).

equity ownership.[4] Due to the dramatic growth in the bonus component in the CEOs' compensation package over the last decade,[5] we expect that further research and more attention will be devoted in the future to the bonus component.

Jensen and Meckling (1976) introduced the principal-agent agency problem. The purpose of incentive schemes is to solve this problem by motivating the agent to take decisions which maximize the wealth of the principal. Jensen and Murphy (1990) explain that managerial actions are not perfectly observable by shareholders; hence, compensation policies should be designed to give the manager incentives to select and implement actions that increase shareholders' wealth. Numerous studies have searched for the optimal contract scheme in an effort to achieve this goal (see, e.g., Ross 1973; Holmstrom, 1979; Bebchuk, Freid, and Walker, 2001; and Zhao, 2008), and have attempted to assimilate compensation practices into the firm's theory (see Baker, Jensen, and Murphy, 1998). In this study, we focus on one component of CEOs' compensation—the bonus —which, as has been demonstrated in Table 1 above, has gained much importance in recent years. Specifically, we focus on the short-term incentive bonus component and its effect on the CEO's risk attitude. Therefore, the purpose of Table 1 is merely to show that in most cases the bonus is larger than the annual salary, implying that the potential bonus presumably plays a crucial role in the CEO's decision-making process. Yet, as most bonus schemes are similar in their structure to call option incentives, our results are quite general and can be extended to include other components in the common incentive packages. For example, Ross

[4] For an empirical analysis of the relation between options and equity incentives with risk-taking policies see, for example, Agrawal and Mandelker (1987), Mehran (1992, 1995), May (1995), Tufano (1996), Berger, Ofek and Yermack (1997), Guay (1999), Dong, Wang and Xie (2010) and many others. For a theoretical analysis of executive stock-based and options incentives see, for example, Acharya, John, Sundaram (2000), Grasselli and Henderson (2009) and Carmona, León, and Vaello-Sebastià, (2011).

[5] Murphy (1999) reports that the bonus component, which in 1992 was smaller than the base salary, increased substantially faster than the base salary during the 90s, a tendency which, according to Table

(2004) who focuses on option-like incentives shows that, similar to the options incentives, the bonus incentive—where the bonus is linked to, say, the accounting earnings—does not necessarily induce the agent to take greater risks (see Ross 2004, p.211), a result which is in contradiction to the common belief.

Does the typical bonus scheme actually motivate the CEO to take excessive risks? Is it generally true that taking greater risks is optimal from the CEO's point of view? Is this generally a false statement, an assertion that is true only for restricted preferences, as suggested by Ross (2004), for the simple case of a convex bonus scheme? Which parameters are controlled by the board of directors and how do they affect the CEO's willingness to take risks? Is there a range within those parameters, which unambiguously motivates the CEO to take excessive risks, and which should therefore be avoided? Finally, does the suggested cap on compensation, which in our specific case implies a cap on the bonus, guarantee that excessive risk will not be taken by the CEO? It is to these questions that we address this study.[6]

There are two main ways to increase a firm's risk exposure: to increase the business risk, i.e. to take on projects with relatively high risk profiles, and to increase leverage. For many firms that operate within a given business, many times it is much simpler to increase risk by changing the financial risk, rather than by changing the firm's business risk. This claim is particularly true as regards the banking industry and financial institutions. Furthermore, the excessive leverage taken by financial

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1, has continued into the 2000s. For a recent analysis of the long-run trends in executive compensation see also Ang, Lauterbach and Schreiber (2002), and Frydman and Saks (2010).

[6] There are two basic approaches for analyzing bonus schemes: a theoretical approach, which suggests an optimal bonus scheme from the stockholders' point of view, and a more practical approach, which analyzes the common bonus schemes. According to the latter approach, one does not intend to completely replace the existing bonus scheme (which is not a simple task to accomplish), but rather to improve it, e.g., by imposing some constraints on the various parameters which determine the bonus. In this study, we adopt this practical approach. Several studies adopt the first approach, which is no less important. Thus, for example, a profound theoretical analysis of optimal bonus schemes and their relations to capital structure can be found in Wang (1997), Douglas (1994, 2006), Guo and Ou-Yang (2006), Noe (2009) and others.

institutions is at the heart of the current crisis. Therefore, in this study we focus on financial institutions and the relation between the typical bonus scheme and the incentive to take extremely large leverage (for the empirical analysis of executive compensation structure in the financial industry, see Ang, Lauterbach and Schreiber. 2002). Analyzing risk-taking via the leverage decision has one important advantage over analyzing risk-taking via projects' selection: In the case of risk-taking via the leverage decision, the effect of leverage on the distribution of return on equity is known; hence, the distribution of the agent's payoff in the case of the typical bonus scheme is also known. Nevertheless, the main conclusions in this study, regarding the bonus scheme and excessive financial risk exposure, are also applied to excessive business risk exposure.

In our model, the CEO is the agent and the firm (stockholders) is the principal, and these two parties may be in conflict regarding optimal risk-taking. Assuming that the agent's goal is to maximize her expected utility (EU) or her Prospect Theory's expected value (EV), which considers her future cash flow, we show that the typical bonus scheme inherently motivates the agent to employ a corner leverage policy: either to employ an extremely large leverage or not to take leverage at all. This result is in line with Ross (2004), who asserts that the incentive scheme does not necessarily induce the agent to take greater risks. However, while Ross assumes risk aversion, we show that his conclusion is also valid within the Prospect Theory framework; namely, preference is not necessarily concave. Moreover, extending the analysis to include the agent penalty for bad performance, we show that in some specific cases, and for some non-pathological preferences, taking excessive risks is optimal from the agent's point of view. This occurs when the cap on the bonus is relatively large (or when there is no cap at all), when the threat of dismissal for poor performance is non-credible, and when the golden parachute is also relatively large. Counter-intuitively, we also find that taking an excessive risk is also optimal from the agent's point of view when the probability of dismissal for poor performance is relatively large, specifically when it is larger than the probability that the return on equity will fall below the risk-free interest rate. Therefore, this tendency substantially intensifies when the interest rates decrease, as occurred in the period preceding the current crisis. Thus, a threat to dismiss the agent for low (but not actually poor) performance induces her to increase risk exposure—quite a surprising result.

Thus, consistent with the findings of Ross (2004), we generally find that the typical bonus incentive schemes do not necessarily induce the agent to take greater risks. This generalizes Ross's (2004) results to include the case of risk-seeking as well as risk-aversion, incentive schemes with payoff functions that are not differentiable in the overall range, and incentive schemes which also consider the penalty, due to a dismissal of the agent for bad performance. However, even after including these features, we also find that in most cases the typical bonus incentive schemes do, in fact, induce the agent to take greater risks. Therefore, we suggest a delicate structure for the bonus scheme, such that there is no incentive to change risk via a change in leverage.

We advocate that a bonus should be paid for management performance, rather than for leverage taking; hence, an ideal bonus scheme should be a "leverage-neutral bonus scheme" (LNBS). Thus, we suggest determining the various bonus components, such that the bonus scheme is an LNBS, or the agent's EU will only be negligibly affected by leverage. These parameters are the target rate of return from which a bonus is paid, the bad performance rate used to dismiss the agent, the cap imposed on the bonus, and the golden parachute. A careful selection of these parameters not only enables the board of directors to affect the agent's decisions, but it can also achieve an LNBS. Having an LNBS, the agent then selects a leverage which is optimal from the stockholders' point of view, hence eliminating agency costs. This result is obtained despite the fact that, with the LNBS approach, the overall structure of the bonus scheme and the total amount of compensation are left to be determined by the board of directors as before, which enables the achievement of the original goals of the bonus.

The current study is closely related to Kanniainen (2000), who shows that under certain conditions a general linear incentive scheme motivates agents with increasing absolute risk aversion to overinvest and thereby to increase agency costs. In this study, we extend these results in several respects. First, we add another dimension to the analysis of the incentive scheme: the agent's implied risk attitude and correspondingly the possibility of excessive risk-taking. Second, we extend the basic bonus linear incentive scheme to include other important and typical components, among them the salary, a cap and a threshold on the bonus, the possibility of dismissal and the effect of the Golden Parachute. Indeed, we show that these common components dramatically affect the agent incentives toward selecting a corner solution. Moreover, the extreme conclusions are not confined to risk averse agents. Finally, we also suggest the LNBS compensation scheme which avoids any distortion in the CEO decisions. Indeed, the current study first proves that the typical bonus scheme generally, albeit not always, induces the agent to take excessive risks. However, we also show that the current short-term bonus scheme may still be used, as long as the bonus parameters are properly determined to achieve LNBS. Thus, we suggest that the typical bonus scheme is still useful, as long as its parameters are properly determined.

The structure of this paper is as follows. Section 1 presents the model. In Section 2, we analyze the effect of the typical bonus scheme on risk-taking and, in particular, on the incentive to take an extremely large leverage. Section 3 presents the concept of LNBS and demonstrates the idea in both expected utility (EU) and Cumulative Prospect Theory (CPT) frameworks. Section 4 concludes. All mathematical proofs are relegated to the Appendices, while in the text we provide the main results, examples, graphical expositions, and intuitive explanations.

1. The Model

We consider an agent who maximizes EU or alternatively maximizes EV, as suggested by the Prospect Theory (see Tversky and Kahneman, 1992). The utility function or the value function is defined by the monetary uncertain outcomes. Thus, if dismissal may damage the agent's reputation, which affects the future income, it is taken into account in our analysis. However, if the agent places a strong emphasis on her good name and self image (see Ellingsen and Johannesson, 2008), or if she adheres to social norms of behavior (see Fischer and Huddart, 2008) even when no monetary consequence is involved, it is not taken into account.

As previously mentioned the current analysis is intended to investigate the most common bonus scheme, based on actual data, rather than other hypothetical and perhaps better bonus scheme. According to Murphy (1999), most CEO compensation packages contain a base salary, an annual bonus linked to short-term accounting performance, special benefits and stock options, and long-term incentive plans (see Table 1). Focusing on the short-term components, we assume that the agent is paid an annual salary and a bonus. The annual salary is independent of the financial results (unless the agent is dismissed); therefore, in our model it may also include the third component presented in Table 1, i.e. other benefits which are also independent of the financial results of that specific year.

In order to introduce the main idea of this paper in a simple way, we ignore taxes. However, essentially the same results are obtained when taxes are incorporated. By not including stock options and long-term incentive plans, we do not claim that these components are not important, but rather only that in this study we analyze the bonus component and its effect on the agent's decisions in isolation.[7] In this respect, our model assumes that the agent is myopic, focusing only on her payoff in the coming year.[8]

The CEOs' annual bonus is typically determined by a formula which is related to the firm's performance. This formula varies across firms and may include several measures.[9] Although firms use a variety of financial and non-financial performance measures, the primary determinant of bonuses is the accounting profits. For example, in Murphy's comprehensive survey, 91% of the firms explicitly use a performance measure based on accounting profits (for the relations between firm's performance and the various performance measures see Hogan and Lewis, 2005). The accounting profits are used to measure performance in various ways; sometimes it is based on dollar terms (e.g. revenues, net income, pre-tax income, operating profits), while at other times it is based on an accounting ratio (e.g. earnings per share—EPS, return on

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[7] It is well-recognized that other things being held constant, the higher the risk taken by the agent, the higher the value of the options given to her. This is because $\partial C / \partial \sigma > 0$, where C is the B&S value of the call option given to the agent (see Black and Scholes, 1973). Thus, albeit not in all cases (see, e.g. Carpenter, 2000, Ross, 2004), this component generally enhances the incentive to take risks (see, e.g., Agarwal and Mandelker, 1987 and Hall and Murphy, 2000). In this study, we show that under certain conditions the agent tends to employ extremely large leverage, even when she solely considers the existing bonus scheme, let alone when the options component is also considered.

[8] For the relation between equity-based compensation and management myopic, see Murphy (2003), for general discussion of myopic decision-making and risk aversion see Benartzi and Thaler (1995, 1999).

[9] According to Murphy (1999), in most cases the multiple measures are "additive" and can essentially be treated like separate plans.

assets and return on equity). As Murphy (1999) noted, most of the firms use a single criterion.

Ellig (2007) provides the following four most common financial-performance criteria: (i) EPS (ii) Return on equity (iii) Return on capital (iv) Return on assets (see p. 326). For example, according to the Fannie Mae executive compensation scheme, senior management reaps financial rewards when the EPS is greater than some growth targets.[10]

Given the above empirical evidence, in this study we employ the accounting rate of return on equity as a criterion for performance. Specifically, we divide the accounting net income by the book value of equity, obtaining a performance index which is frequently employed by banks and financial institutions. This performance criterion is identical to criterion (ii), return on equity, in the common case where the book value of equity is employed. Moreover, our results are the same when net income (i.e. without any division) or when criterion (i), EPS, is employed. This is because in all these criteria the performance measure is net income, and the division by one, by the book value of equity, or by the number of shares, serves only as a scaling procedure which is needed to establish the target rate in dollars, in percent or in EPS terms. Thus, our analysis and the results are essentially the same for all these performance criteria, as well as for any other criterion which measures performance

[10] Specifically to Fannie Mae, three components of compensation depended directly on reaching EPS targets: (1) an Annual Incentive Plan (AIP), under which executives and other managers earned bonuses; (2) a Performance Share Plan that granted stock to senior executives based on three-year performance cycles; and (3) the EPS Challenge Grant, a program for all employees which tied the award of a substantial amount of stock options to the doubling of EPS from 1998 to 2003 (see, Federal Housing Finance Agency – FHFA, 2006. Report of the Special Examination of Fannie Mae - May 2006, p. 55).

according to accounting net income divided by some accounting figure related to the firm's equity.[11]

Under the typical bonus scheme, no bonus is paid until a threshold performance, called "the target rate", has first been achieved. A bonus is paid for achieving a rate of return which is higher than the target rate, and there is typically a cap on the paid bonus. The range between the threshold and this cap is called "the incentive zone" (see Murphy, 1999). According to Murphy (1999), the most common payout method (for all but financial companies) is the "80/120" plan, where no bonus is paid unless performance exceeds 80% of the performance standard, and bonuses are capped once performance exceeds 120% of the performance standard. Other common combinations include 90/110, 95/100, 50/150, 80/110, 90/120, and 80/140 plans (for a general analysis of pay-performance relations, see e.g. Jensen and Murphy, 1990). Finally, Murphy's comprehensive survey shows that in 13% of the firms, there is no cap where this phenomenon is more common in the financial sector, which is at the center of the current economic crisis.

To implement the incentive zone, we assume that when the rate of return on equity is below the target rate of return—which is determined by the board of directors at the beginning of the year—no bonus is paid. When the rate of return on equity is above this target rate of return, the bonus is given as a fraction of the difference between the realized rate of return on equity and the target rate of return on equity. Generally, there is also a maximum rate of return which determines a cap on the maximum bonus. Thus, the incentive zone is bounded between the target rate of return and the maximum rate of return. Below this zone no bonus is paid, within this zone

[11] If market values are employed or when total assets are considered in the denominator of the performance measure, adjustments to the model are required.

the bonus increases together with rate of return, and above this zone a fixed maximum bonus is paid.[12]

To complete the relations between the agent's compensation scheme and performance, it is assumed that if the realized rate of return is below a certain minimum rate, e.g. the risk-free interest rate or zero, the agent is dismissed, an event which entails a loss of wealth for the agent (For general analysis of forced CEO dismissal see Huson, Parrino and Starks, 2001, and Hori and Osano, 2009). This loss can be relatively large if, for example, no other firm will hire the agent. Alternatively, the loss can be relatively small, e.g. in the case of an elderly agent who intends to retire soon, regardless of the firm's financial results. Let us now turn to the formal model according to which we analyze the optimal leverage from the agent's point of view.

Notations

In the analysis below we employ the following notations:

$U(\cdot)$ – The agent's utility or value function, which is a function of monetary uncertain outcomes. The function $U(\cdot)$ is assumed to be differentiable, strictly increasing and to satisfy $\lim_{x\to\infty} u'(x) < \infty$;

E – Book value of equity at the beginning of the year;

D – Book value of debt;

$A = E + D$ – Book value of total assets;

$L = D/E$ – Accounting leverage employed;

Z – The firm's net income before taxes (a random variable);

$\rho = Z/A$ – The pure equity firm's rate of return on total assets (a random variable);

[12] Our suggested linear increase of the bonus in the incentive zone is similar to Murphy's (1999) example of a typical bonus scheme (see Figure 5 in his study). However, he also notes that a convex or concave incentive zone may also be applied.

r – Interest rate on the firm's debt, which is assumed to be constant, and which may be larger than the risk-free interest rate.

$R_e = (Z - rD)/E = Z/E - rD/E = (Z/A)(A/E) - rD/E = \rho + (\rho - r)L$ – The levered firm's rate of return on equity (a random variable);[13]

R_T – Target rate of return on equity. For rate of return greater than this rate, a bonus is paid;

$R_{\min}$ – Minimal rate of return on equity. For rate of return below this rate, the agent is dismissed;

$R_{\max}$ – The rate of return on equity for which the maximum bonus is paid. For a rate of return above this rate, the agent is paid a fixed maximum bonus. Thus, this value imposes a cap in dollar terms on the bonus paid;

K – The present value of the agent's all-inclusive loss in a case of dismissal. If a golden parachute is given in a case of dismissal, it is included in K.

W_0 – The agent's annual base salary;

W_B – The agent's bonus (a random variable);

W – The total payoff to the agent (a random variable);

M – The agent's initial wealth;

γ – The bonus percentage figure of the excess profit (see blow), where $0 < \gamma < 1$.

Obviously, we always assume that $R_{\min} < R_T < R_{\max}$ and that $r < R_T < R_{\max}$. However, regarding the relation between $R_{\min}$ and r, we separately analyze two

[13] We ignore corporate tax. However, with corporate tax, T, we have $\rho_T = \rho(1-T)$ and $R_{e,T} = (1-T)(Z - rd)/E = (1-T)(Z/A)(A/E) - (1-T)rD/E = (1-T)\rho(E+D)/E - (1-T)rD/E = (1-T)\rho + (1-T)(\rho - r)D/E = \rho_T + (\rho_T - r)D/E$, where ρ_T and $R_{e,T}$ stand for the after-tax return on total assets and return on equity, respectively. Thus, with corporate tax we obtain the same structure for return on equity, implying that corporate tax changes the magnitude of the results, but not the basic features.

possible cases: $R_{\min} < r$ and $R_{\min} \geq r$. Finally, in the rest of the paper, the term *rate of return* relates to the accounting rate of return, which is relevant to the bonus determination. Using these notations and emphasizing that ρ, R_e and W are random variables, we can then analyze the effect of the agent's bonus scheme on her tendency to take excessive risks.

The agent's annual compensation is composed of two components: an annual base salary (W_0) and an annual bonus (W_B). Using the above notations, the agent bonus for a given year is determined as follows:

$$W_B = \begin{cases} 0 & if\ R_e \leq R_T \\ \gamma E(R_e - R_T) & if\ \ R_T < R_e \leq R_{\max} \\ \gamma E(R_{\max} - R_T) & otherwise \end{cases}, \tag{1}$$

where R_T and $R_{\max}$ are usually determined by the board of directors (see Murphy, 1999) and presumably affected by the past average profitability in the relevant industry.[14] Note that as R_e, $R_{\max}$ and R_T are percentage figures and E is the dollar book value of equity, W_B is given in dollar terms. Finally, if no cap is imposed on the bonus $R_{\max}$ in Eq. (1) is replaced by infinity and the third range no longer exists.

The bonus payoff scheme given in Eq. (1) is similar to call options given to the agent. Indeed, according to Ross (2004) a convex bonus scheme similar to that in Eq. (1) does not necessarily induce the agent to increase risk-taking. However, after including the fixed base salary which is independent of performance and the penalty for bad performance, we obtain below a payoff function which is not differentiable in several points, a strict departure from the option-like payoff scheme.

[14] Occasionally, the bonus is defined in terms of total profit, rather than in terms of excess profits. In addition, it is common to have a step function incentive in which γ is determined corresponding to various profit intervals (see Murphy, 1999 and Ellic, 2007). For simplicity's sake, in the analysis above we employ Eq. (1)'s bonus scheme, while bearing in mind that other schemes of the same type essentially yield the same results.

The bonus is only one component affecting the agent's payoff and, correspondingly, her decisions. If a severe loss occurs, i.e. the firm's rate of return falls below a certain threshold, $R_{\min}$, the agent is dismissed (and although this is not formally specified in the standard contract, we assume that the agent knows that such an event may occur). In the case of such an event, the agent incurs a net present value all-inclusive loss of –*K*. Namely, –*K* reflects the net present value of any monetary loss, including the loss due to the damage to reputation (which decreases her chances of finding another well-paying job), the cost of looking for another job, and so forth. Although in the case of a very generous golden parachute –*K* may be positive, it is assumed that this is an exceptional case, and we generally assume that –*K* is negative.

Considering all these payoff components, the agent's next year's compensation is given as follows:

$$W = \begin{cases} -K & \text{if } R_e \le R_{\min} \\ W_0 & \text{if } R_{\min} < R_e \le R_T \\ W_0 + \gamma E(R_e - R_T) & \text{if } R_T < R_e \le R_{\max} \\ W_0 + \gamma E(R_{\max} - R_T) & \text{otherwise} \end{cases} . \quad (2)$$

Thus, for normal performance ($R_{\min} < R_e \le R_T$) the agent's salary is W_0, for relatively good performance ($R_e > R_T$) the agent is also rewarded with a bonus and paid $W_0 + \gamma E(R_e - R_T)$ with a cap of $W_0 + \gamma E(R_{\max} - R_T)$, where for $R_{\max} = \infty$ in Eq. (2), we have a model suitable for a bonus scheme with no cap on the bonus. Hence, the incentive zone is within the range of $R_T < R_e \le R_{\max}$. Finally, for very poor performance ($R_e \le R_{\min}$) the agent faces a loss of –*K* (where *K* is the present value of all future income lost in the case of dismissal after adding any portion of the annual salary, W_0, and the golden parachute which is paid despite the dismissal).

Note that if bankruptcy occurs, it is assumed that the loss to the agent is also –*K*. However, we make the reasonable assumption that the board of directors dismisses the agent for what is termed "bad" performance, yet performance which is still better than the one inducing bankruptcy; hence, a possible bankruptcy does not change our results. Thus, for simplicity's sake, and without loss of generality, it is assumed no bankruptcy, yet a dismissal of the agent is possible.[15] Consequently, in the relevant range of leverage the firm borrows at a fixed interest rate, *r* (the cost of debt), which may be equal to or larger than the risk-free rate.

In what follows we analyze the agent's EU in terms of the distribution of the rates of return on an unlevered firm, because the distribution of rates of return on the equity of a levered firm changes with the decision variable—the leverage that we analyze in this paper. Suppose that the agent's utility function is given by $U(W)$. One can analyze the agent's leverage decision either according to the Expected Utility (EU) framework of von-Neumann & Morgenstern (1953) or that of the Cumulative Prospect Theory (CPT) of Kahneman and Tversky (1992). In EU analysis, the utility is defined on total wealth; therefore, one should add the agent's initial wealth, *M*. In the CPT framework, the analysis is in terms of the *change* in wealth and the initial wealth, *M*, should be ignored.

Let us begin with the EU framework, after which we will demonstrate that the results are very similar under the CPT framework. The agent's EU is given by:

$$\begin{aligned} E[U(W)] = &\int_{-\infty}^{R_{\min}} U(M-K)f(R_e)dR_e + \int_{R_{\min}}^{R_T} U(M+W_0)f(R_e)dR_e + \\ &\int_{R_T}^{R_{\max}} U[M+W_0+\gamma E(R_e-R_T)]f(R_e)dR_e + \int_{R_{\max}}^{\infty} U[M+W_0+\gamma E(R_{\max}-R_T)]f(R_e)dR_e, \end{aligned} \tag{3}$$

[15] For a general analysis of the impact of bankruptcy codes on the firms' capital-structure choices see Acharya, John, and Sundaram (2008).

where $f(R_e)$ is the density function of the random rate of return on equity. When we switch to the CPT framework, M is eliminated (M = 0) and $U(\cdot)$ is no longer the utility function, but rather the value function.

The accounting rate of return on equity is given by:

$$R_e = \rho + (\rho - r) \times D / E = \rho + (\rho - r) L, \tag{4}$$

where ρ is the accounting rate of return on total assets and $L \equiv D/E$ stands for the firm's degree of leverage.[16] Denoting by $f_\rho(\rho)$ the density function of ρ and using Eq. (4), Eq. (3) can be rewritten as follows:[17]

$$\begin{aligned} E[U(W)] = & \int_{-\infty}^{\frac{R_{\min}+rL}{1+L}} U(M-K) f_\rho(\rho) d\rho + \int_{\frac{R_{\min}+rL}{1+L}}^{\frac{R_T+rL}{1+L}} U(M+W_0) f_\rho(\rho) d\rho + \\ & \int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} U\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T]\} f(\rho) d\rho + \int_{\frac{R_{\max}+rL}{1+L}}^{\infty} U[M+W_0+\gamma E(R_{\max}-R_T)] f(\rho) d\rho, \end{aligned} \tag{5}$$

Or equivalently,

$$\begin{aligned} E[U(W)] = & U(M-K) F_\rho\left(\frac{R_{\min}+rL}{1+L}\right) + U(M+W_0)\left[F_\rho\left(\frac{R_T+rL}{1+L}\right)\right. \\ & \left. - F_\rho\left(\frac{R_{\min}+rL}{1+L}\right)\right] + \int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} U\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T]\} f_\rho(\rho) d\rho + \\ & + U[M+W_0+\gamma E(R_{\max}-R_T)] \times \left[1 - F_\rho\left(\frac{R_{\max}+rL}{1+L}\right)\right], \end{aligned} \tag{6}$$

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[16] Eq. (4) is similar to Modigliani and Miller's (1958) Proposition II with the distinction that in this study we employ book values, rather than market values of debt and equity.

[17] For two random variables with the following linear relation: $y = ax + b; a > 0$, we have the following well-known relations: $f_y(y)|_{y=ax+b} = \frac{1}{a} f_x\left(\frac{y-b}{a}\right)$ and $dy = adx$. Therefore, we have, $\int f_y(y)dy|_{y=ax+b} = \int \frac{1}{a} f_x\left(\frac{y-b}{a}\right)dy = \int \frac{1}{a} f_x\left(\frac{y-b}{a}\right)adx = \int f_x(x)dx$ where the limits of the integral are adjusted to the change in variable (see, e.g, Bierens, 2005, p. 136). In our case, $R_e = \rho + (\rho - r)L = \rho(1+L) - rL$, hence $a = (1+L) > 0$. Treating R_e as y and ρ as x and

where f and F are the distribution density function and the cumulative distribution function of the firm's rate of return on total invested assets, ρ, respectively. In the coming sections, we use Eq. (6) to analyze the optimal leverage policy from the agent's point of view. We show that the bonus parameters must be determined within certain bounds to prevent the agent's tendency to take excessive risks.

2. The Bonus Scheme and the Agent's Leverage Policy: Extreme Leverage Policy

In this section, we analyze the effect of the various bonus parameters on the agent's optimal leverage policy and, in particular, on her tendency to employ either zero leverage or extremely large leverage. Assuming that these two extreme policies represent over-conservatism or, more severely, excessive risk-taking from the shareholders' point of view, this analysis guides us toward a bonus scheme which avoids these two extreme policies or at least makes such policies less likely. Our suggestion for LNBS is discussed in Section 3.

It is well-known that the larger the employed leverage, the larger the fluctuations in the rate of return on equity. Therefore, in the case of a bad year, relatively low rates of return are recorded which, in turn, may result in a dismissal of the agent. Therefore, one may be tempted to believe that a relatively large rate of return from which the agent is dismissed (i.e. a relatively large $R_{\min}$) encourages the agent to decrease leverage because the probability of dismissal is relatively large, and apparently leverage increases this probability. Surprisingly, however, in Proposition 1 below we show that just the opposite holds true: a relatively large $R_{\min}$ encourages the agent to take excessive risks by employing an extremely large leverage. Moreover,

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recalling that $\rho = (R_e + rL)/(1+L)$, Eq. (5) is obtained from Eq. (3), when the limits of the integral are changed as required by the relation between R_e and ρ.

this counter-intuitive result remains intact under any agent's preference and under any penalty level, *K*.

Proposition 1: *Let* $R_e, \rho, R_{\min}, R_{\max,} R_T, r, K, W$ *and* L *be defined as above. If* $R_{\min} \geq r$ *(i.e.,* $R_{\min}$ *is relatively large, hence the probability of dismissal is relatively large), then* $\partial E[U(W)]/\partial L > 0 \forall L \geq 0$ *regardless of the assumed preference and the size of the loss, K. Therefore, the higher the leverage, the better off the agent is. Formally, for every* $L_2 > L_1$, $F(W/L_2)$ *dominates* $F(W/L_1)$ *by First degree Stochastic Dominance (FSD),*[18] *where* $F(W)$ *is the cumulative distribution function of the agent's payoff, W. This FSD remains intact for any bonus cap which is greater than the target rate.*

Proof: In this proof we analyze the distributions $F(R_e/L_2)$ and $F(R_e/L_1)$ and the corresponding distributions $F(W/L_2)$ and $F(W/L_1)$. We have to prove that for $L_2 > L_1$, $F(W/L_2) \leq F(W/L_1) \forall W$ (i.e. policy L_2 dominates policy L_1 by FSD), which implies that $E[U(W/L_1)] \leq E[U(W/L_2)]$ for all non-decreasing preferences (see Footnote 18). To prove Proposition 1, we need to calculate the probability of two events. First, let us calculate the probability of the event asserting that no bonus is paid, i.e. the probability of $R_e < R_T$. This probability is given by:

$$F(R_T/L) = \Pr(R_e \leq R_T) = \Pr[\rho(1+L) - rL \leq R_T] = \Pr[\rho \leq (R_T + rL)/(1+L)].$$

[18] For two cumulative distributions *F* and *G*, *F* dominates *G* by FSD, implying that *F* is preferred by all non-decreasing preferences, $U(\cdot)$. Formally,

$F(X) \leq G(X) \forall X \Leftrightarrow E_F[U(X)] \geq E_G[U(Y)] \forall U; U' > 0$. For more details, see Hanoch and Levy (1969), Hadar and Russel (1969), Rothschild and Stiglitz (1970) and Levy (2006).

As by assumption $R_T > R_{\min} \geq r$, it is straightforward to show that the larger the L, the smaller this probability is.[19] Hence, if $L_1 < L_2$, $F(R_T / L_2) < F(R_T / L_1)$.

Bearing in mind the above result, let us now calculate the probability of a dismissal of the agent. Using the same procedure as given above it is easy to show that the probability of a dismissal is given by: $F(R_{\min} / L) = \Pr[\rho \leq (R_{\min} + rL)/(1+L)]$. As by assumption of Proposition 1 $R_{\min} \geq r$, this probability decreases with an increase in L.[20] Thus, the higher the leverage the smaller the probability of a dismissal.

Using the probabilities of these two events and the relation between the distribution of R_e and W, we now show that $F(W / L_2)$ dominates $F(W / L_1)$ by FSD. First, note that $F(R_e / L_2)$ differs from $F(R_e / L_1)$ only by the degree of leverage. Therefore, these two cumulative distributions intersect only once at the well-known break-even point $R_e = r$. At this point, the leverage has no effect on the rate of return on equity.

The two cumulative distributions, corresponding to L_1 and L_2, are presented in Figure 1.a, where $F(R_e / L_1)$ is steeper than $F(R_e / L_2)$.

<< Insert Figure 1 >>

As it has been proven above that the probability of dismissal is larger with L_1 than with L_2, we can conclude that $P_1 > P_2$. Figure 1.b presents the distributions of the agent's payoff under the two leverage policies: $F(W / L_1)$ and $F(W / L_2)$. As regards

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[19] As by assumption $R_T > r$, we have $\frac{\partial}{\partial L}\frac{(R_T + rL)}{(1+L)} = \frac{(r - R_T)}{(1+L)^2} < 0$ and the larger the L, the smaller the cumulative probability. $F(R_T / L)$.

$R_e < R_{\min}$ a loss of $-K$ occurs, and as $P_1 > P_2$, we observe that $F(W/L_2)$ is located below $F(W/L_1)$ in the range $R_e < R_{\min}$, which corresponds to the range $W < W_0$, given in Figure 1.b. As the probability of not receiving a bonus (i.e. the probability that $W < W_0$) is smaller with L_2 than with L_1, we have $F(W < W_0 / L_2) < F(W < W_0 / L_1)$; hence, $P_2^* < P_1^*$. As regards $R_e > R_T$, which corresponds to the range $W > W_0$, the bonus is always larger with L_2 than with L_1;[21] $F(W/L_2)$ is also located below $F(W/L_1)$ in this range.

Combining the results corresponding to all ranges of R_e, we obtain that $F(W/L_2) < F(W/L_1)$ for the entire range of *W*, as is shown in Figure 1.b. Therefore, as regards $R_{\min} \geq r$ and $L_2 > L_1$, leverage policy L_2 dominates leverage policy L_1 by FSD. Q.E.D.

The above FSD dominance can be explained by means of Figure 1.a: by increasing leverage, the distribution rotates around point *r*. As both $R_{\min}$ and R_T are located to the right of point *r*, both the probability of a dismissal and the probability of not receiving a bonus decrease (see Figure 1.a); hence, EU increases regardless of the preference and regardless of the size of *K*. The reason for this counter-intuitive result is that with $R_{\min} \geq r$ the probability of a dismissal is always smaller with L_2 than with L_1; hence, regardless of the size of *K*, the agent is better off by increasing leverage.

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[20] As $\frac{\partial}{\partial L}\frac{(R_{\min}+rL)}{(1+L)} = \frac{(r-R_{\min})}{(1+L)^2} \leq 0$, for $R_{\min} \geq r$, the probability of a dismissal $F(R_{\min}/L)$ decreases with an increase in *L*. Of course, for $R_{\min} < r$ this derivative is strictly positive.

[21] Because in this range, the larger the leverage, the larger R_e – see Eqs. (2) and (4) for the range $R_e > R_T$.

It is interesting to note that Proposition 1's strong result of FSD differs from Ross's (2004) result, which shows that the effect of a simple convex bonus scheme on the agent's risk-taking behavior is ambiguous. This is because Proposition 1 deals with a specific risk, due to leverage, which also specifically determines a particular distribution of the payoff to the agent and because we add the loss, due to a possible dismissal, to the payoff. Thus, while Ross deals with a general option-like payoff which, in his model, is also twice differentiable, in our case there is a particular payoff which, among other differences, is not convex and not twice differentiable over most of the relevant range.

To prevent the agent from taking excessive risks, the board of directors should determine $R_{\min}$ to be smaller than the interest rate, i.e. $R_{\min} < r$. However, in Proposition 2 below we show that $R_{\min}$ should not be "too small"; otherwise, the same tendency to employ an extremely large leverage policy will be obtained.

Proposition 2. *Let* $R_e, \rho, R_T, R_{\max,}, r, W$ *and L be defined as above. In addition, assume that the agent is not penalized for bad performance, implying that the agent's income never falls below W_0 (i.e.* $\Pr(R_e \le R_{\min}) = 0$*). Then,* $\partial E[U(W)] / \partial L > 0 \forall L \ge 0$ *regardless of the assumed preference. Hence, like in Proposition 1, for every* $L_2 > L_1$*,* $F(W/L_2)$ *dominates* $F(W/L_1)$ *by FSD and correspondingly, the higher the leverage, the better off the agent is. This FSD remains intact for any bonus cap which is greater than the target rate.*

Proof: Like in Proposition 1, the cumulative distribution corresponding to L_1 and L_2, where $L_2 > L_1$, rotates around point *r* (see Figure 1.a). Hence, also in this case we observe that the larger the leverage, the smaller the probability that the agent will not receive a bonus. Therefore, here we also have $P_2^* < P_1^*$. However, as the probability of dismissal is zero, regardless of the leverage policy, we also have that $P_2 = P_1 = 0$. Thus, $F(W/L_2) \le F(W/L_1)$ for any W with at least one strict inequality, and leverage policy L_2 dominates leverage policy L_1 by FSD. Q.E.D.

Figure 1.c illustrates the results of Proposition 2. As can be seen, as regards $\Pr(R_e \le R_{\min}) = 0$, $F(W/L_2)$ is always below $F(W/L_1)$; hence, the L_2 leverage policy dominates the L_1 leverage policy by FSD. Unlike Proposition 1, Proposition 2 is very intuitive. When no penalty is involved, increasing leverage always increases the probability of receiving the bonus without any negative implications. Hence, the agent always has the incentive to increase leverage, thereby increasing her EU.

Discussion

From the above two Propositions it emerges that $R_{\min}$ should be carefully selected: to avoid an extreme leverage policy a lower and upper bounds on $R_{\min}$ must be set. It is worth mentioning that implementing these bounds in practice is not an easy task. This is because generally $R_{\min}$ is not formally determined in the agent's contract, but is rather left as an implicit issue (for explicit versus implicit CEO contracts, see Gillan, Hartzell and Parrino, forthcoming). However, according to Proposition 2, a credible threat must be determined either formally or informally; otherwise, the agent tends to take an extremely large leverage.

While the result of Proposition 2 has an intuitive explanation, the result of Proposition 1 and, in particular, the irrelevancy of K is counter-intuitive, probably because in practice it is more common to have $R_{\min} < r$, rather than $R_{\min} \ge r$. Yet, it is very likely to have the complementary and problematic case where $R_{\min} \geq r$ in two cases, one of which is very relevant to the current crisis. The first case is when the average rate of return on equity in a given industry is substantially larger than *r*. In this case, achieving a rate of return which is just above *r* may be considered as a severe failure and a good reason for dismissing the agent. To illustrate this claim, suppose that the mean rate of return on the equity in a given industry is, say, 15% and the interest rate is, say, 4%. Achieving a rate of return of, say, 5% in a given year where most firms belonging to the same industry earn on average, say, 15%, may be a sufficient cause to dismiss the agent.

The other case may occur when the board of directors has determined $R_{\min}$ such that $R_{\min} < r$ at the beginning of year (or when the agent's contract is signed). However, a sharp drop in the interest rates during the year may induce a transformation to the complement regime where $R_{\min} \geq r$. This case is quite reasonable in years of dramatic changes in interest rates. In fact, a drop in the interest rate and low interest rates characterized the period which preceded the current crisis. Whatever the reason, , Proposition 1 shows that having a situation where $R_{\min} \geq r$ is undesirable because it motivates the agent to take excessive risks by employing an extremely large leverage.

Figure 2 illustrates the agent's hypothetical EU as a function of leverage for the two cases where $R_{\min}$ is not within the bounds given in Propositions 1 and 2.

<< Insert Figure 2 >>

Figure 2 reveals that when $R_{\min} \geq r$ (Graph 1) and when $\Pr(R_e \leq R_{\min}) = 0$ (Graph 2), EU always increases with leverage (i.e. $\partial E[U(W)]/\partial L > 0 \forall 0 \leq L$).[22] This property is general and does not depend on preference, on *K*, or on any other parameter, but rather relies on the FSD. Moreover, these results do not depend on the cap, given in terms of rate of return on equity, as long as it is greater than the target rate, which is a trivial requirement. Therefore, when $R_{\min}$ is not within the correct bounds, decreasing the bonus cap—as has been recently suggested—does not change the agent's tendency to take excessive risks.

Following Propositions 1 and 2, hereafter we suggest that any reasonable bonus scheme that avoids the FSD should fulfill the conditions $R_{\min} < r$ and $\Pr(R_e \leq R_{\min}) > 0$. Therefore, in the rest of the paper we assume that these two conditions hold true without repeating this assumption again. In Proposition 3—which is most relevant to our paper—we show that the bonus incentive scheme does not necessarily induce the agent to increase risk-taking, even with non-convex incentive schemes, and even when the payoff function is not differential. Thus, Proposition 3 generalizes Ross's (2004) result to also include the typical case of non-convex incentive schemes and payoff functions which are not differentiable.

Proposition 3*: Let* $R_e, \rho, R_{\min}, R_{\max}, R_T, r, K$ *and L be defined as above. If* $R_{\min} < r$ *and* $\Pr(R_e \leq R_{\min}) > 0$*, for two leverage policies L_1 and L_2, where* $L_2 > L_1$*, there is no FSD of the larger leverage policy over the smaller leverage policy regardless of the*

[22] The same result holds true for the case of strict equality $R_{\min} = r$. Substitute $R_{\min} = r$ in Eq. (8) yields, $\partial E[U(W)]/\partial L = \gamma E\int_a^b U'\{M + W_0 + \gamma[\rho + (\rho - r)L - R_T]E\}(\rho - r) f_\rho(\rho) d\rho$, where $a = (R_T + rL)/(1+L)$ and $b = (R_{\max} + rL)/(1+L)$, which is always positive.

selected control variables. Moreover, there is no Second degree Stochastic Dominance (SSD)[23] *of the larger leverage policy over the smaller leverage policy. However, the board can determine the control variables that guarantee that the smaller leverage policy dominates the larger leverage policy by SSD.*

Proof: As R_T is located to the right of point r, and as the cumulative distribution rotates around point r (see Figure 3.a), like in Proposition 1 we have that the larger the leverage, the smaller the probability that the agent will not be paid a bonus (of course with the trivial assumption that $R_T > r$). Therefore, in this case we also have $P_2^* < P_1^*$ (compare Figures 1 and 3). However, regarding the probability of a dismissal of the agent, we have $F(R_{\min} / L) = \Pr[\rho \leq (R_{\min} + rL)/(1+L)]$. Thus, for $R_{\min} < r$, unlike the case presented in Proposition 1, here this probability increases together with the leverage, L, as $\frac{\partial}{\partial L}\frac{(R_{\min} + rL)}{(1+L)} = \frac{(r - R_{\min})}{(1+L)^2} > 0$. Namely, in contrast to Proposition 1, in this case: $P_2 > P_1$. Q.E.D.

The intuitive explanation for the above result is as follows: As in the previous case where $r \geq R_{\min}$, likewise in the case of $R_{\min} < r$ the two distributions, $F(R_e / L_1)$ and $F(R_e / L_2)$, cross each other only once, at the break-even point $R_e = r$, a point where the leverage has no effect on the rate of return on equity. The result asserting that P_2 is greater than P_1 stems from the fact that by assumption $R_{\min}$ is located to the left of the rotation point r, and that the two distributions cross each other only once, at point r (see Figure 3.a).

[23] For two cumulative distributions F and G, by SSD we have, $\int_{-\infty}^{X}[G(t) - F(t)]dt > 0 \forall X \Leftrightarrow E_F[U(X)] \geq E_G[U(X)]$ for all concave preferences, $U(\cdot)$.

The result that there is no SSD in Proposition 3 is in line with Ross's (2004) result, which shows that the effect of a simple convex bonus scheme on the agent's risk-taking is ambiguous. Namely, despite the different settings of the two models, when the bonus parameters are properly determined, as in Proposition 3, the bonus scheme in both cases does not necessarily induce the agent to increase risk-taking. However, as in our case we also consider the payoff to the agent in the case of a penalty, due to bad performance (not included in Ross's model), the opposite result may hold true. Namely, under certain bonus scheme parameters, the bonus scheme can induce the agent to decrease risk-taking by decreasing leverage (see the last part of Proposition 3). Let us elaborate.

Figure 3.b reveals the two distributions of the agent's payoff corresponding to two leverage levels, for the same distribution of R_e as in Figure 1, with the exception that this time we relate to the case where $R_{\min} < r$.

<< Insert Figure 3 >>

As $P_2 > P_1$ and $P_2^* < P_1^*$, the two distributions $F(W/L_1)$ and $F(W/L_2)$ cross each other and there is no FSD. It is straightforward to show that there is no Second degree Stochastic Dominance (SSD) of the larger leverage policy over the smaller leverage level policy.[24]

However, it is possible to have SSD of the smaller leverage policy over the larger leverage policy. This occurs when *K* is relatively large and $R_{\max}$ is relatively small, a case where area $|A|$ in Figure 3.b is larger than area *B*. In this case, when only two leverage policies, L_1 and L_2 ($L_1 < L_2$), are considered any risk-averse agent will choose the smaller leverage policy.[25] Thus, the board of directors can choose $R_{\max}$, such that L_1 dominates L_2 by SSD, hence avoiding the relatively larger leverage under consideration. However, this $R_{\max}$ does not guarantee that L_1 dominates L_3 by SSD, where $L_1 < L_2 < L_3$. In other words, each value of $R_{\max}$ can be tailor-made for a given pair of leverage policies, L_1 and L_2, and one cannot determine one $R_{\max}$ to fit all leverage policies. To show that $R_{\max}$ for which L_1 dominates L_2 by SSD does not guarantee that L_1 dominates L_3 one example is sufficient. The following example illustrates this case (see also Figure 6.a).

An example

Let us illustrate the above results with a very simple example. Suppose that $R_{\min} = 0\% < r = 5\% < R_T = 10\% < R_{\max} = 50\%$, $\gamma E = \$200$, $W_0 = \$5$, $M = \$30$, $K = -\$30$ and the utility function is linear, i.e. $U(W) = W$ (note that shifting to a risk-averse preference does not change the main idea given here, as is proven in Proposition 4 below). Let the possible outcomes of ρ be -5%, 0%, 5% and 30%, all with an equal probability of ¼. According to these assumptions, when no leverage has been employed, the agent's EU is given by,

$$E[U(W)] = \frac{1}{4}(0) + \frac{1}{4}(35) + \frac{1}{4}(35) + \frac{1}{4}[35 + (0.3 - 0.1) \times 200] = \frac{145}{4}.$$

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[24] There is no SSD because $\int_{-K}^{W_0} [F(W/L_1) - F(W/L_2)]dw < 0$; hence, $F(W/L_2)$ does not dominate $F(W/L_1)$ and $\int_{-K}^{\infty} [F(W/L_2) - F(W/L_1)]dw < 0$ (because leverage increases the mean return); hence, $F(W/L_1)$ does not dominate $F(W/L_2)$ (see also Figure 3).

[25] It is possible that $F(W/L_1)$ dominates $F(W/L_2)$ by SSD. This case occurs when area $|A|$ is larger than area B (see Figure 3.b), which takes place when $R_{\max}$ is close to R_T. In this case, we have $\int_{-K}^{W} [F(W/L_1) - F(W/L_2)]dw > 0 \forall W$; hence, policy L_1 dominates policy L_2 by SSD.

Employing a modest leverage of 20% ($L = 0.2$), Eq. (4) yields the distribution of R_e to be -7%, -1%, 5%, 35%, all with equal probability of ¼. Recalling that $R_T = 10\%$ and $R_{\min} = 0\%$, the agent's EU with $L = 0.2$ is given by,

$$E[U(W)] = \frac{1}{4}(0) + \frac{1}{4}(0) + \frac{1}{4}(35) + \frac{1}{4}[35 + (0.35 - 0.1) \times 200] = \frac{120}{4} < \frac{145}{4}.$$

In other words, by shifting from $L = 0$ to $L = 0.2$, the agent's EU decreases.

Suppose that one increases the leverage to $L = 1$. For $L = 1$ the distribution of R_e is given by, -15%, -5%, 5%, 55%, all with equal probability of ¼. In this case, the agent's EU is given by,

$$E[U(W)] = \frac{1}{4}(0) + \frac{1}{4}(0) + \frac{1}{4}(35) + \frac{1}{4}[35 + (0.5 - 0.1) \times 200] = \frac{150}{4} > \frac{145}{4}.$$

Thus, by shifting from $L = 0$ to $L = 1$, the agent's EU first decreases and then increases, such that for $L = 1$, it is larger than the EU corresponding to $L = 0$.

According to Proposition 3 and the numerical example, it is clearly shown that generally when $R_{\min} < r$, and the probability of a dismissal is positive, the EU is not a monotonic function of the leverage and there is no FSD of a certain leverage policy over another. Therefore, under these conditions the optimal leverage policy depends on the agent's preference (see Ross, 2004). Furthermore, unless some specific parameters are determined, there is no SSD. Thus, to find the agent's exact optimal leverage policy one needs to know the agent's preference, $U(\cdot)$, and the distribution of returns, $f_\rho(\rho)$ —quite a discouraging result.

Nevertheless, as we shall show in the next section, even without this knowledge, tighter general conclusions can be made regarding the optimal bonus scheme, such that a corner solution is not optimal from the agent's point of view.

Specifically, we show that the board of directors can determine the bonus parameters to have an LNBS in which the agent's EU is unaffected, or negligibly affected, with leverage. With this optimal set of control variables, leverage does not affect the agent's EU; therefore, she may employ the leverage which is optimal from the stockholders' point of view and the excessive risks and agency costs are eliminated.

3. Neutral Leverage Bonus Scheme (LNBS)

When no FSD and SSD exist, as demonstrated in the above example, the agent's EU may be a non-monotonic function of leverage.[26] Let us now turn to the conditions of the various parameters, which guarantee non-monotonicity. From Eq. (A5) in Appendix A, we have:

$$\frac{\partial E[U(W)]}{\partial L} = [U(M-K) - U(M+W_0)] f_\rho \left(\frac{R_{\min}+rL}{1+L}\right)\left[\frac{(r-R_{\min})}{(1+L)^2}\right] + \int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} U'\{M+W_0+\gamma[\rho+(\rho-r)L-R_T]E\}\gamma E(\rho-r) f_\rho(\rho)d\rho \equiv P+B, \tag{8}$$

where P stands for the Penalty marginal effect and B stands for the Bonus marginal effect. By definition, $P \le 0$ (because $M-K < M+W_0$) and $B \ge 0$ (because all the arguments in B are positive). If $P+B=0$ for a given leverage policy, but not for all leverage policies, we have a non-monotonic EU function. Ideally, the board of directors determines the bonus parameters to obtain LNBS, namely a bonus scheme for which $P+B \cong 0 \forall L$. In such a case, the agent who is indifferent to the leverage policy presumably employs the leverage policy which is optimal from the shareholders' point of view. However, determining the bonus parameters to obtain LNBS is not an easy task, as is shown below.

[26] Formally, the first order condition in Eq. (6) is where Eq. (8) is equal to zero, i.e. $P+B=0$.

Eq. (8) reveals that the marginal effect of increasing leverage on the agent's EU depends on several parameters. However, regarding the parameters determined by the board of directors, two cases should be avoided: setting $R_{\min}$ to be close to r would generally lead to an extremely large leverage (as P approaches zero and B is positive), while setting $R_{\max}$ to be close to R_T (or setting γ to be close to zero) would lead to zero leverage (as P is negative and B approaches zero). Thus, to avoid an extreme leverage policy, the upper bound on $R_{\min}$ should be lower than the bound given by Proposition 1 (i.e. $R_{\min} << r$) and there must be upper and lower bounds on the incentive zone ($R_{\max} - R_T$).

These results are quite intuitive. When, as a result of an increase in leverage, the change in the probability of a dismissal is small relative to the change in the probability and the size of the bonus, the utility of the latter always outweighs the disutility of the former. In contrast, when the change in the probability of dismissal is sufficiently larger than the change in the probability and the size of the bonus with the increase in leverage, the disutility of the former always outweighs the utility of the latter.

Proposition 3, accompanied by the numerical example, reveals that considering two possible leverage levels, one can determine a cap which prevents the agent from taking the larger leverage policy of the two policies under consideration.[27] Thus, this cap may prevent all agents from preferring a given relatively large leverage

[27] To obtain the SSD dominance given in Proposition 3, one can technically increase R_T, rather than decrease $R_{\max}$. However, there is a critical difference between these two methods which makes the latter, rather than the former, more practical. While decreasing $R_{\max}$ decreases the potential bonus in the case of very good performance, when the agent is already entitled to the maximum bonus, increasing R_T affects the potential bonus in the case of normal performance, when the agent is entitled to a small bonus or even no bonus at all. Therefore, a risk-averse agent is harmed much more by increasing R_T. Thus, if the goal is to control the agent's risk appetite, it is preferable to decrease her utility by changing $R_{\max}$, rather than changing R_T.

policy over a given lower leverage policy. However, as has been shown in the numerical example above, this given cap may not prevent the agent from preferring an even larger leverage policy than the two given leverage policies under consideration. In such a case, a cap on the bonus may result in a leverage which is even larger than the one the cap originally intended to prevent. Thus, Proposition 3 is rather discouraging because a cap may not rule out extremely large leverage policies. Therefore, one should focus on the more general goal of determining the cap such that it prevents taking all extremely large leverage policies, and not only avoiding a selection of a specific given leverage policy. Actually, the goal is to choose a cap, such that the agent is indifferent to the leverage policy.

Indeed, we show below that a careful selection of the cap may produce an LNBS in which the agent's EU is unaffected by leverage. We first numerically demonstrate how such a goal can be achieved and then explain the results in technical terms. However, we stress at the outset that although we use a specific numerical example in this case, the existence of such a cap—which produces an LNBS—is the rule rather than the exception, as is proven at the end of Appendix A.

Figure 4 illustrates a hypothetical agent's EU, as a function of leverage for various values of $R_{\max}$.[28]

<< Insert Figure 4 >>

Figure 4 shows that when $R_{\max}$ is relatively small (Graph 1), the EU always decreases with leverage and the optimal leverage policy is zero leverage. Similarly, when $R_{\max}$ is relatively large (Graph 2), there may be a strong incentive for the agent to increase leverage, thereby increasing her EU. However, continuously reducing $R_{\max}$, we reach

[28] While in Figures 4 and 5 we assume that ρ is normally distributed, we verified that the results are very similar when we assume other distributions, such as the uniform and lognormal distributions.

a point where the EU is almost flat, indicating that we have obtained the desired result: from a certain size of small leverage, leverage becomes irrelevant, as it hardly affects the agent's EU (Graph 3). Thus, in this specific example $R_{\max} = 57.2\%$ yields an LNBS or almost an LNBS.

Let us now explain why, in the general case—by continuously reducing the cap—an almost flat line, as is presented in Graph 3, is obtained. If by decreasing the cap, Graph 2 in Figure 4 was simply shifted downward, the flat line would not have been obtained. However, we claim that by decreasing the cap the curve shifts down with a rotation, such that a flat line is generally obtained. Namely, by decreasing the cap the EU declines faster at points of larger leverage in comparison to points of smaller leverage. To see this more clearly, let us go back to Figure 3.b. It is obvious that the larger the leverage, the larger the area *B*. By decreasing the cap, the area *B* is reduced without affecting area *A*. As for a relatively large leverage, the size of area *B*, which is eliminated by reducing the cap, is larger in comparison to the small leverage case. The EU is reduced by a higher amount in the larger leverage case, hence the rotation in Graph 2 in Figure 4 (additional examples of LNBS are presented below).

Let us now turn back to Eq. (8) for a more formal explanation as regards obtaining an LNBS. Generally, one may need a relatively large *L* to guarantee that $P \to 0$ and $B \to 0$ [29]. However, one can determine $R_{\max}$, such that $P + B \cong 0 \forall L > L^*$ (see Eq. (8)), where L^* is relatively small, hence $\partial E[U(W)] / \partial L \cong 0 \forall L > L^*$. Namely, for a given set of parameters to have an LNBS, one should determine $R_{\max}$, such that $B \cong -P$, hence $B + P \cong 0$. The general existence of such $R_{\max}$ is based on the fact that

[29] It is easy to see that $\lim_{L \to \infty} P = 0$ and $\lim_{L \to \infty} B = 0$ (see Appendix A).

both P and B in Eq. (8) converge at the same rate of order 2 to zero with L (for a proof, see the end of Appendix A).

One may argue that these general results regarding the cap are quite intuitive and therefore probably employed by all firms. However, this is not the case as Murphy's (1999) study shows that in his sample 13% of the firms did not have any cap on the bonus, and in others the cap may be too large to bind. Thus, it seems that the typical bonus schemes occasionally violate these results; therefore the agent whose EU looks like Graph 2 in Figure 4 has an incentive to take excessive risks by increasing leverage. In contrast, our suggested LNBS or almost LNBS may avoid this tendency towards excessive risk-taking.

In the previous paragraph, we provide one example of LNBS; below we add a few more examples of LNBSs when the following scenarios are also considered: the agent's degree of risk aversion, the effect a golden parachute, and the possibility that the agent makes decisions according to Cumulative Prospect Theory, rather than EU. These examples illustrate how by carefully determining $R_{\max}$, one can obtain $B + P \cong 0 \forall L \geq L^*$, where L^* is relatively small.

3.1 The bounds on the golden parachute

According to Proposition 3, in order to prevent the agent from employing extremely large leverage the board of directors may not only select R_T and $R_{\max}$, but may also affect the loss involved with a dismissal, *K*. As we show below, affecting *K* may be an effective tool to prevent a zero leverage policy; however, it may also intensify the agent's tendency to take excessive risks by employing an extremely large leverage.

The present value of the loss in the case of a dismissal, K, depends on the specific case. For example, K can be very small if the agent finds anther job within a short time period. On the other hand, K can be very large for an agent whose reputation is badly damaged and who cannot find a job at a level similar to the current one. Similarly, as mentioned earlier, for a young agent, K is usually much larger than for an old agent who is about to retire. Nevertheless, the board of directors may affect K by providing a golden parachute when a dismissal occurs. Indeed, the agent's compensation generally also includes a golden parachute, which means that the agent is entitled to a large extra payoff upon leaving her job. Moreover, in many cases the agent is entitled to this payoff, or at least to a portion of it, even if she is dismissed, a situation which has incited a great deal of criticism during the current crisis. As we show below, this criticism stands on solid ground. Specifically, in terms of our model, paying a relatively large golden parachute in the case of dismissal decreases K. Therefore, if this golden parachute is sufficiently large, it may once again encourage the agent to take excessive risks by employing extremely large leverage.

Intuitively, decreasing K by means of a generous golden parachute, other things remaining constant, increases the agent's EU.[30] More importantly to leverage policy, decreasing K increases P in Eq. (8). Thus, a modest golden parachute which is paid in the case of dismissal may weaken the agent's tendency toward zero leverage policy. However, if the golden parachute is sufficiently large, it will further intensify her tendency towards an extremely large leverage. Thus, the effect of the golden parachute depends on the specific parameters and, in particular, on the size of K. Therefore, using a large golden parachute to avoid the tendency of the agent toward

[30] This can also be seen from the differentiation of Eq. (6) with respect to K, which yields: $\frac{\partial E[U(W)]}{\partial K} = -U'(M-K)F_{\rho}(\frac{R_{\min}+rL}{1+L}) < 0$.

zero leverage is more reasonable in the case of a young agent with a large *K*. In contrast, a large golden parachute in the case of an elderly agent will probably intensify her tendency towards extremely large leverage.

Figure 5 demonstrates the effect of the golden parachute for a given set of parameters.

<< Insert Figure 5 >>

As can be seen from Figure 5, for a relatively small loss in the case of dismissal the larger the leverage, the higher the EU (see Graph 1); therefore, the agent always has a strong incentive to increase leverage. In contrast, for a relatively large potential loss, the larger the leverage, the lower the EU (see Graph 2); therefore, the agent always has a strong incentive to decrease leverage. However, by determining *K* in our specific example to be $K = 0.24M$, we obtain an LNBS. Thus, for a given set of parameters the board of directors can determine a golden parachute, which affects *K*, such that LNBS is created. Obviously, this optimal golden parachute varies with the problem parameters.

3.2 The effect of risk aversion

In the previous sections, we have shown that two cases should be avoided or at least mitigated: the case where the agent always has an incentive to increase leverage, and the case where the agent always has an incentive to decrease leverage. In Proposition 4, which is given below, we can see that when risk aversion is introduced the tendency to increase leverage does not necessarily disappear, while the tendency to decrease leverage further intensifies. Therefore, the basic problems of the existing bonus schemes, which are demonstrated above as regards risk-neutrality, also exist as regards risk-aversion.

Proposition 4*: Let $R_e, \rho, R_{\min}, R_{\max}, R_T, r, K$ and L be defined as above. Consider two levels of leverage: L_1 and L_2, where $L_1 < L_2$. Assume also that $R_{\min} < r$ and* $\Pr(R_e \le R_{\min}) > 0$.

1. If L_1 is preferred to L_2 by a risk-neutral agent*, then such a preference, a fortiori, also holds true for a risk-averse* agent*.*

2. If L_2 is preferred to L_1 by a risk-neutral agent*, it does not imply that this preference necessarily exists for a risk-averse* agent*.*

3. Risk aversion generally does not imply that extreme leverage is not optimal from the agent'*s point of view.*

(for a proof see Appendix B)

The results of Proposition 4 are very intuitive. If the increase in the loss, in the case of dismissal from increasing the leverage, outweighs the corresponding increase in the potential bonus for a risk-neutral agent, then this also holds true *a fortiori* for a risk-averse agent. However, if the increase in the potential bonus outweighs the corresponding increase in the loss, in the case of dismissal for a risk-neutral agent, then the preference of a risk-averse agent depends on various parameters. In some cases, a risk-averse agent may prefer to increase leverage, while in other cases the opposite holds true.

To illustrate the results of Proposition 4, Figure 6.a shows two hypothetical EUs corresponding to a risk-neutral preference: where $U(W) = W$, and a risk-averse preference where $U(W) = W^{1-\alpha}/(1-\alpha)$ and $R_{\max} = 70\%$. According to Proposition 4, when Graph 1 decreases, Graph 2 must also decrease. However, when Graph 1 starts increasing, Graph 2 still decreases reaching its minimum, in our specific example, at

$L^{\min} = 0.4$ in comparison to $L^{\min} = 0.3$ with Graph 1. In both cases, with $R_{\max} = 70\%$ the agent is better off when employing an extremely large leverage.

<< Insert Figure 6 >>

Can one obtain an LNBS in the face of risk aversion? And if so, how does this solution relate to the risk-neutral case solution? First, similar to the risk-neutral case, one may also obtain, in the risk aversion case, the LNBS for a given set of parameters by gradually decreasing $R_{\max}$ until $P + B \cong 0$ in Eq. (8). Indeed, Figure 6.b shows a hypothetical EU for the same preferences and the same parameters as in Figure 6.a, only this time $R_{\max}$ is reduced to obtain an LNBS. Thus, for $R_{\max} = 50\%$ and $R_{\max} = 53.5\%$ we obtain an LNBS for both risk-neutral and risk-averse agents, respectively. As expected, Figure 6.c shows that if $R_{\max}$ is further reduced—to say, $R_{\max} = 45\%$ —then the agent's optimal leverage policy is to employ zero leverage.

The effect of risk aversion on the optimal cap makes sense, as the utility value of area *B* (see Figure 3) is smaller with risk aversion; hence, one needs to increase the cap to obtain the same effect as that which is obtained with a risk-neutral preference. Thus, although a cap on the bonus does not guarantee that EU is not increased with leverage (see Figure 6.a), determining the optimal $R_{\max}$ guarantees that for $L \geq L^*$, $\partial E[U(W)] / \partial L \cong 0$, hence an LNBS is obtained. Notably, although the optimal cap with risk aversion is larger than the optimal cap with risk neutrality, the difference is quite small. On the positive side, it indicates that the results are not very sensitive to the assumed preference.

Finally, in practice to calculate the cap which is required to obtain an LNBS with a risk-neutral preference, one need only know the distribution of rate of the return on equity (f_ρ), since the rest of the parameters are known. As the cap in this case is smaller than the cap in the case of a risk-averse agent, then this value can serve as a benchmark in the common case where the agent's preference is unknown.

3.3 The agent's expected value: Cumulative Prospect Theory

Nowadays, Kahneman and Tversky's (1979) Prospect Theory (PT) and its modified version, Kahneman and Tversky's (1992) Cumulative Prospect Theory (CPT), are commonly employed to explain people's choices. Indeed, the experimental evidence reveals that people do not make decision as predicted by the EU, but rather as predicted by the CPT. We demonstrate below that our results are essentially the same when the agent's preference is determined by the CPT framework. Formally, this can easily be seen in the general case by substituting the value function given in Eq. (10) below in Eq. (6).

The main features of the CPT, which are relevant to our study, assert that people make decisions based on change of wealth, rather than total wealth, and are characterized by a preference called "value function" with risk seeking for $X \le 0$ and risk aversion for $X > 0$. Moreover, the segment corresponding to $X \le 0$ is steeper, i.e. loss aversion prevails. This value function is given by:

$$V(X) = \begin{cases} X^{\alpha} & X > 0 \\ -\lambda(-X)^{\beta} & X \le 0 \end{cases}. \tag{9}$$

Tversky and Kahneman (1992) provide the following parameter estimates of the value function: $\alpha = \beta = 0.88$ and $\lambda = 2.25$. To capture the loss aversion characteristic, Benartzi and Thaler (1995) employ the bi-linear value function where $\alpha = \beta = 1$ and $\lambda = 2.25$. Note that in the CPT framework the agent's initial wealth is irrelevant as X is defined in terms of the change in wealth. Thus, we substitute $V(X)$ from Eq. (9) instead of $U(W)$ into Eq. (6) where X stands for the agent's payoff (i.e. $M = 0$).

Figures 7.a, 7.b and 7.c show the agent's EV, $E[V(X)]$ as a function of the employed leverage for K&T's value function and for the bi-linear value function.

<< Insert Figure 7 >>

Basically, the results are very similar to those obtained for the EU analysis. With $R_{\max} = 80\%$ (Figure 7.a) the agent's optimal leverage policy is to employ the maximum possible leverage. However, with $R_{\max} = 71\%$ and $R_{\max} = 64.3\%$ (Figure 7.b) we obtain an LNBS for the two cases, respectively. Namely, as expected, the higher the risk aversion, the larger the $R_{\max}$, which is required to obtain the LNBS. Thus, as demonstrated in Figure 7, changing the parameters of the value function changes the LNBS cap, but it does not change the basic features of the analysis.

4. Concluding Remarks

Employing an extremely large leverage and greediness are mentioned as two reasons for the current crisis in the world's financial markets. In this study, we analyze the effect of the short-term component of the CEO's (the agent) compensation on her motivation for excessive leverage taking. We focus on the annual bonus component, which presumably affects the CEO's risk decision, because it is relatively large, typically even larger than the annual salary. We do not suggest an optimal bonus scheme, which may induce major changes that are difficult to implement, but rather suggest some limits regarding the parameters of the common bonus schemes, which avoid major distortion in risk-taking by the CEOs.

While with a twice differentiable incentive scheme function, Ross (2004) shows that the folklore asserting that convex incentive schemes induce the agent to take greater risks is false, we show that Ross's result holds true even with more realistic payoff functions, which are not differentiable over the entire range. However, the non-differentiable payoff function with an additional negative payoff in the case of bad performance also induces various other results, with even First degree Stochastic Dominance of one leverage policy over the other.

The common perception is that agents should be compensated for efficient managing, but not according to random performance induced by leverage-taking, which may randomly increase or decrease the rate of return on equity. Thus, an optimal bonus scheme should be a “leverage-neutral bonus scheme” (LNBS). Analyzing the typical bonus scheme, we find the following results, which are robust to the selected distribution of rate of return on total assets, as well as to the agent’s preference and framework, i.e., whether she maximizes expected utility or expected value function, as suggested by the Prospect Theory.

1. In the most general case, there is no Second degree Stochastic Dominance (SSD) of one leverage policy over the other, which is in line with Ross’s assertion that the effect of a convex incentive scheme on risk-taking is ambiguous.

2. Extending the model to incorporate the agent’s possible loss, due to a dismissal, we find that when the risk of dismissal is relatively large, the agent has an incentive to increase, rather than to decrease, the firm’s risk exposure. Hence, the agent tends to increase the leverage. Specifically, when the minimum rate of return from which the agent is dismissed is larger than the risk-free interest rate, the agent’s expected utility increases with the increase in leverage. This counter-intuitive result does not depend on the agent’s preference or on the distribution of returns, as from the agent’s point of view in this case a larger leverage policy dominates a smaller leverage policy by First degree Stochastic Dominance (FSD). Moreover, this result is unaffected by the amount of the agent’s loss in the case of dismissal.

3. When the threat of dismissal for poor performance is non-credible, i.e. the agent envisions a zero probability of being dismissed, the asymmetry in the payoff from the agent's point of view is very large; hence, a larger leverage policy always dominates a smaller leverage policy by FSD. This, once again, implies excessive risk-taking.

4. When the threat of dismissal is credible, the bonus is paid for the rate of return, which is above some target rate, and there is no cap on the bonus; generally, the agent's expected utility is not a monotonic function of leverage. Hence, for any two levels of leverage, there is no dominance of one level over the other. Usually, but not always, this function may have a minimum; beyond this minimum the larger the leverage, the better off the agent implying, once again, excessive risk-taking.

Given the above conclusions, the most important result of this paper is that the board of directors can determine a bonus scheme which is an LNBS or almost an LNBS. This could be done as follows: First, establish a credible threat for dismissal; the rate of return from which the agent is dismissed for poor performance should be below the risk-free interest rate. Second, a cap on the bonus must be imposed. For most distributions and for most preferences, either in an expected utility framework or in the Prospect Theory framework, there is a cap which produces an LNBS or almost an LNBS. Of course, this optimal cap is also a function of the other bonus parameters (e.g. the size of the golden parachute), and therefore should be simultaneously determined with the rest of the parameters. However, the board can fix the other parameters and then reduce or increase the cap, until an LNBS is obtained. Fortunately, there is a relatively low sensitivity of the cap which produces an LNBS to the agent's degree of risk aversion.

According to the above procedure, the agent who faces an LNBS has no incentive to take excessive leverage; hence, she presumably chooses the optimal leverage from the stockholders' point of view. With the suggested procedure, agency costs corresponding to the bonus scheme vanish and the threat to the stability of the economy, due to leverage taking, is not excessive. Importantly, this positive result is obtained, despite the fact that the overall structure of the bonus and the total amount of compensation are left to be determined as before, which makes it possible to achieve the original goals of the bonus. In contrast, failing to follow this procedure, excessive risk is optimal from the agent's point of view and a high level of leverage is predicted, as has occurred during the recent financial crisis.

Finally, a few words of caution are called for. First, although the theory regarding the determination of the cap on the bonus is relatively simple, implementing it may be difficult and one needs a very experienced board of directors to select the optimal cap. Despite this difficulty, in this study we warn against fatal mistakes which can be easily avoided by the board. Secondly, we assert that either an LNBS or almost an LNBS can be achieved. In the case where such a policy is "almost" achieved, the agent may choose not to use any leverage at all. While this policy may be non-optimal for the stockholders, at least it does not jeopardize the firm. Moreover, it can be easily shown that to avoid this case the board can also give the agent stocks; hence, her relatively low preference for not taking the tax advantage of leverage from the stockholders' point of view will disappear.

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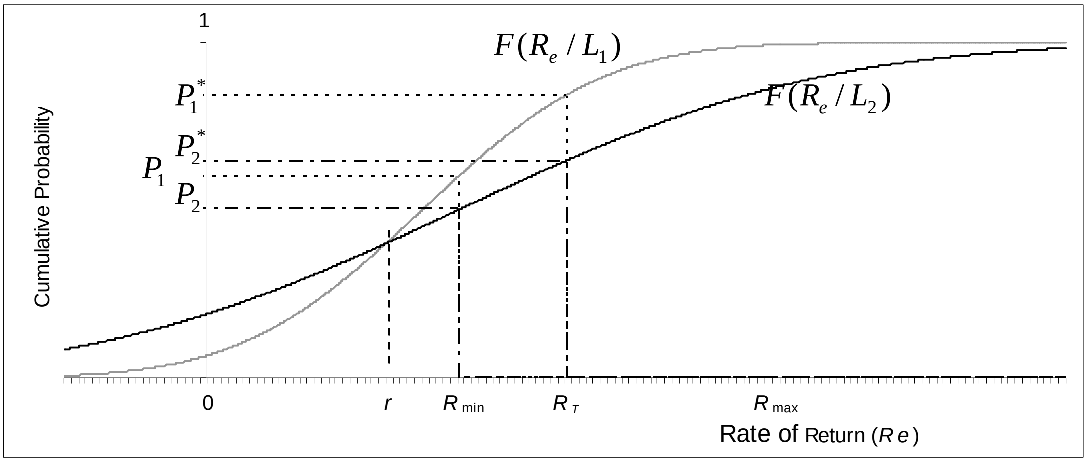
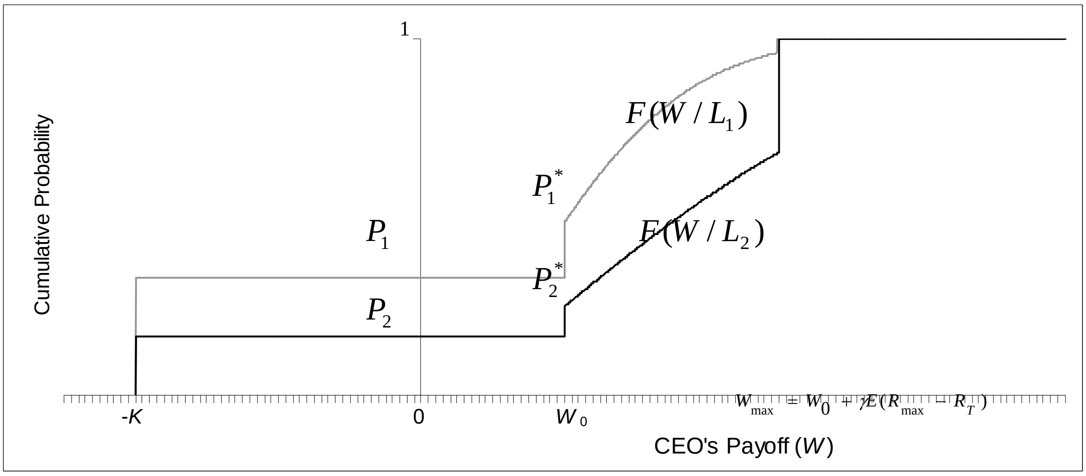


1.b



1.c

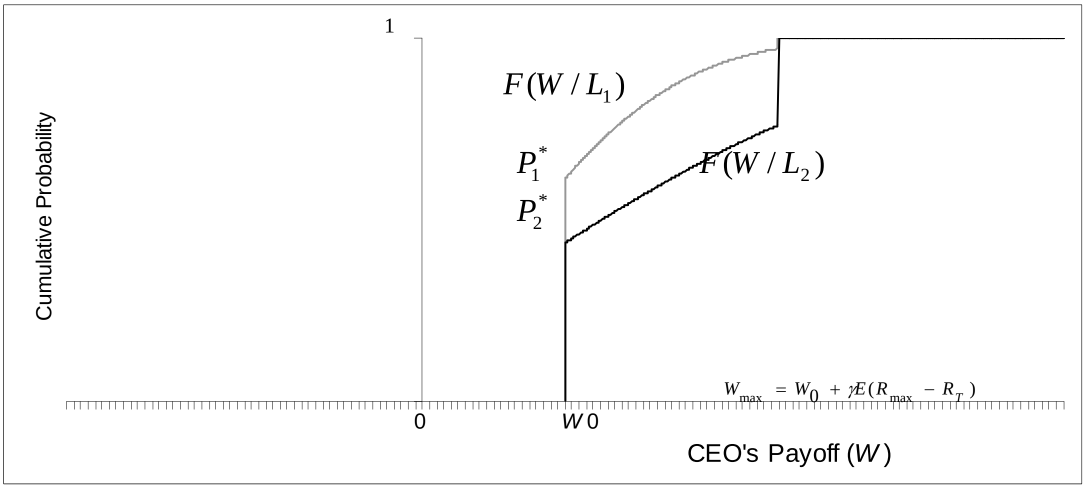


Figure 1. Leverage policy and the agent's expected utility where $R_{\min} \geq r$ or $\Pr(R_e < R_{\min}) = 0$

The figures depict the cumulative distribution of the firm's rate of return on equity, $F(R_e)$ (Figure 1.a) and the cumulative distribution of the agent's wealth, $F(W)$ (Figures 1.b and 1.c) for two leverage policies, L_1 and L_2 where $L_1 < L_2$. In Figures 1.a and 1.b $R_{\min} \geq r$ and in Figure 1.c $\Pr(R_e < R_{\min}) = 0$. Hence, $F(W / L_2)$ dominates $F(W / L_1)$ by First degree Stochastic dominance (FSD).

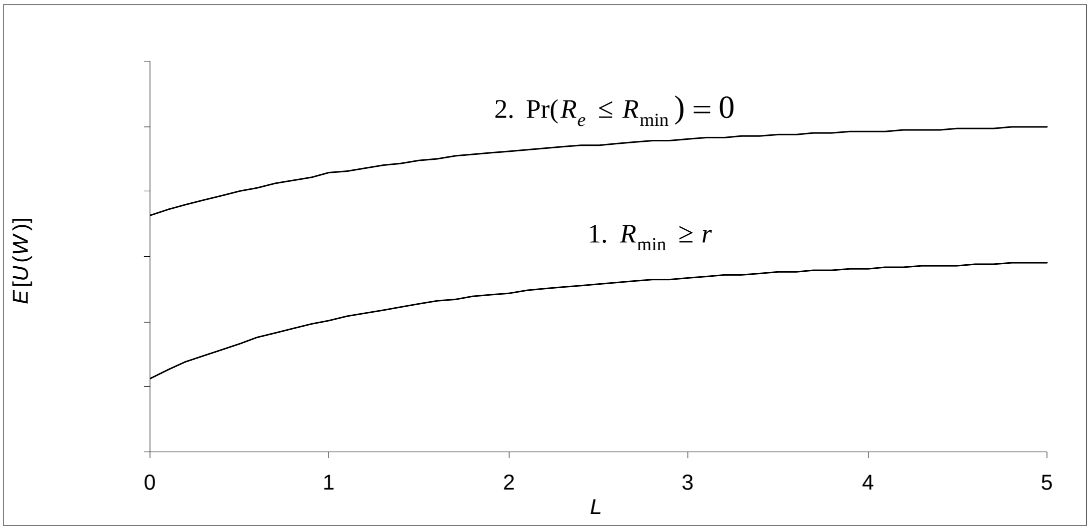
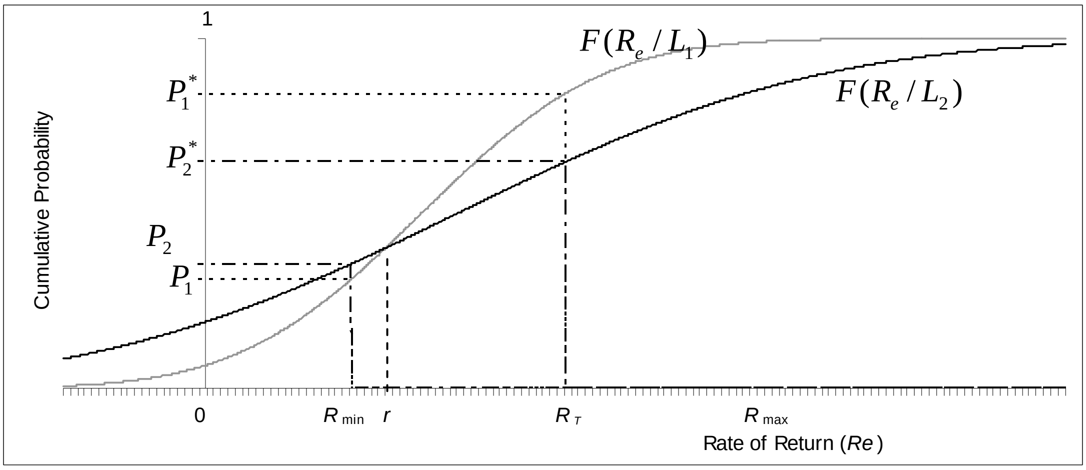


Figure 2. The effect of improperly determined dismissal rate on the optimal leverage policy

The figure depicts the agent's expected utility, $E[U(W)]$, as a function of leverage for the two cases in Figure 1: (i) When $R_{min} \geq r$ (Graph 1) (ii) When R_{min} is unbinding, $\Pr(R_e \leq R_{min}) = 0$ (Graphs 2). In both cases, $\partial E[U(W)]/\partial L \geq 0 \forall L \geq 0$ and therefore the agent's optimal leverage policy is to employ the maximum possible leverage. The figure is drawn with the following parameters: $U(W) = W$, $M = \$10{,}000{,}000$, $W_0 = \$1{,}000{,}000$, $\gamma E = \$4{,}000{,}000$, $K = \$1{,}500{,}000$, $r = 5\%$, $R_{max} = 60\%$, $R_{min} = 10\%$ (Graph 1) or $R_{min} = -\infty$ (Graph 2) and ρ is normally distributed: $\rho \sim N[10\%, 20\%]$

3.a



3.b

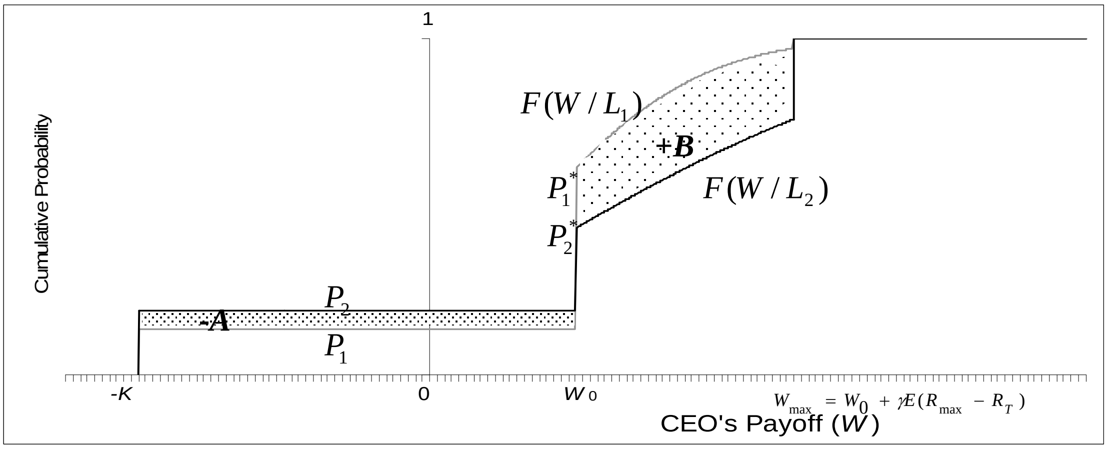


Figure 3. Leverage policy and the agent's expected utility where $R_{min} < r$ and $\Pr(R_e < R_{min}) > 0$

The figures depict the cumulative distribution of the firm's rate of return on equity, $F(R_e)$ (Figure 2.a) and the cumulative distribution of the agent's wealth, $F(W)$ (Figure 2.b) for two leverage policies, L_1 and L_2 where $L_1 < L_2$. The minimal rate of return, for which the agent is dismissed, is within the desired bounds, i.e. $R_{min} < r$ and $\Pr(R_e < R_{min}) > 0$; hence, no stochastic domination of one leverage policy over another exists.

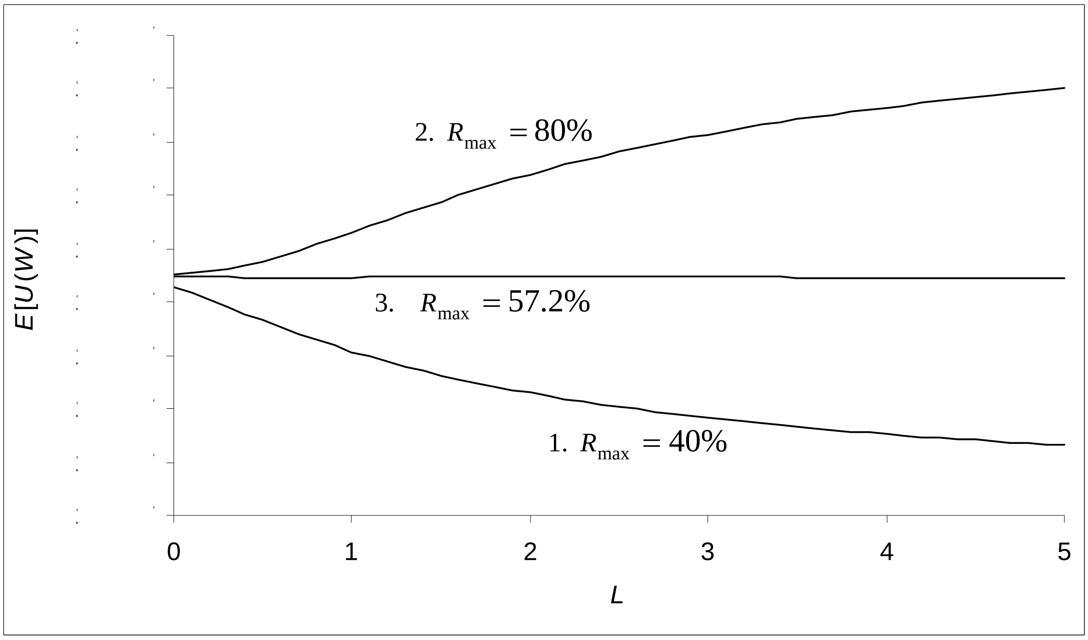


Figure 4. The bonus cap effect on leverage policy

The figure depicts the agent's expected utility, $E[U(W)]$, as a function of leverage, for various values of $R_{\max}$. Graphs 1 and 2 show the cases where $R_{\max}$ is relatively small or relatively large, respectively. Graph 3 shows the case where $R_{\max}$ is carefully determined to obtain a leverage-neutral bonus scheme (LNBS). The figure is drawn with the following parameters : $U(W) = W$, $M = \$10{,}000{,}000$, $W_0 = \$1{,}000{,}000$, $\gamma E = \$4{,}000{,}000$, $K = \$1{,}500{,}000$, $r = 5\%$, $R_{\min} = -20\%$ and ρ is normally distributed: $\rho \sim N[10\%, 20\%]$. The values for $R_{\max}$ are 40%, 80% and 57.2%, respectively.

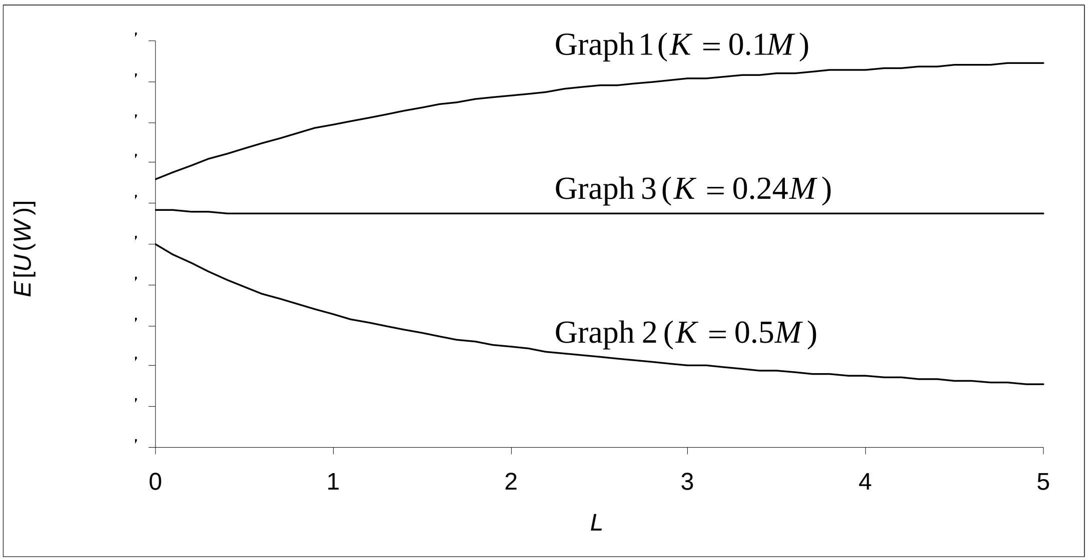
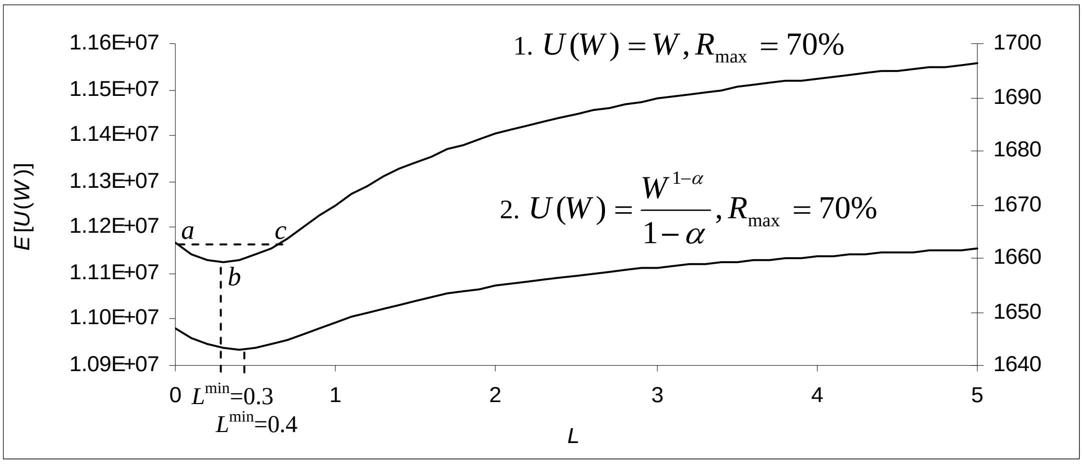


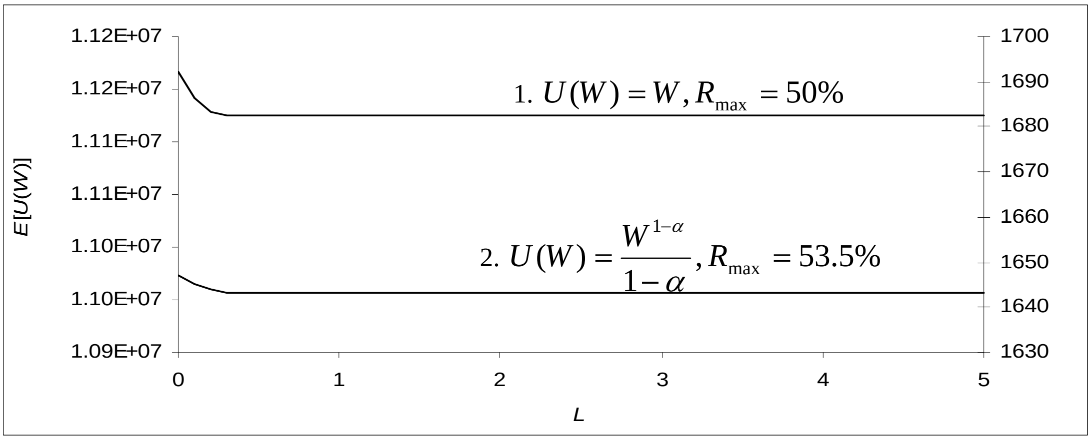
Figure 5. The golden parachute effect on leverage policy

The figure depicts the agent's expected utility, $E[U(W)]$, as a function of leverage for three present values of the loss, K, in the case of a dismissal. When the loss is relatively large (Graph 1) and when the loss is relatively small (Graph 2) the agent has a strong incentive to increase and decrease leverage, respectively. However, if carefully determined, the golden parachute can mitigate these undesirable tendencies as demonstrated in Graph 3. The figure is drawn with the following parameters: $U(W) = W$, $M = \$10{,}000{,}000$, $W_0 = \$1{,}000{,}000$, $\gamma E = \$4{,}000{,}000$, $r = 5\%$, $R_{\min} = -20\%$, $R_{\max} = 0.6$ and ρ is normally distributed: $\rho \sim N[10\%, 20\%]$.

6.a



6.b



6.c

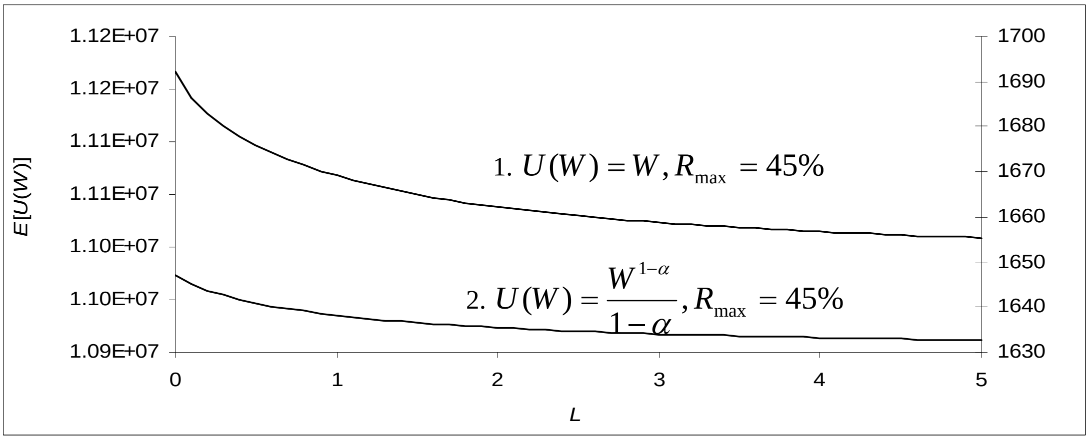
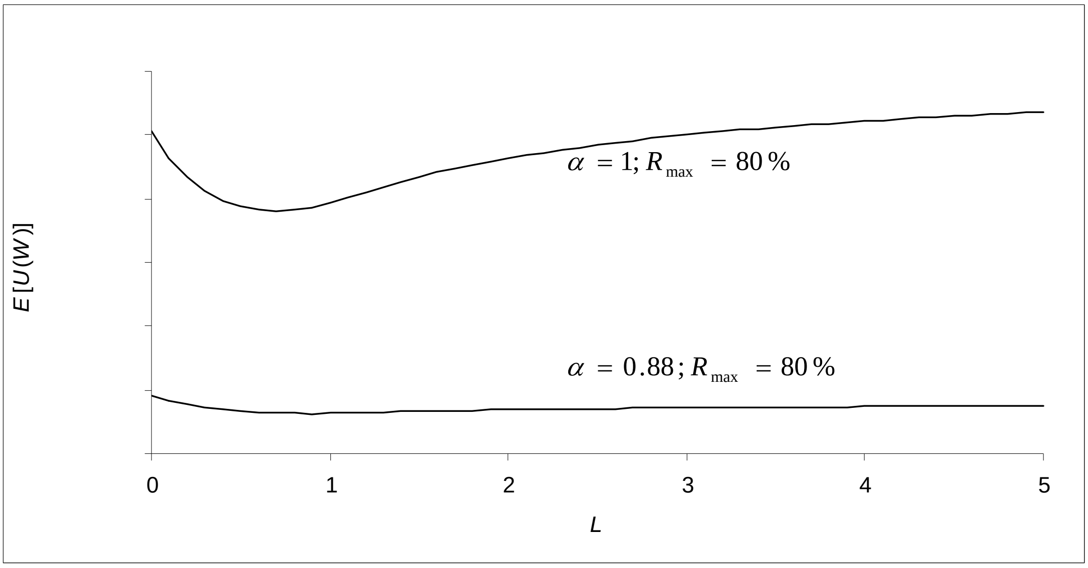


Figure 6. The effect of risk-aversion on the optimal leverage policy

Figure 6.a depicts the agent's expected utility, $E[U(W)]$, as a function of leverage for two preferences: risk neutrality, $U(W) = W$, (Graph 1) and CRRA, $U(W) = W^{1-\alpha}/(1-\alpha)$, (Graph 2). Figures 6.b and 6.c depict the same two cases only $R_{\max}$ is reduced once to obtain a leverage-neutral bonus scheme (LNBS) (Figure 3.b), and once so that it will be below the LNBS's value (Figure 6.c). The figures are drawn with the following parameters: $M = \$10{,}000{,}000$, $W_0 = \$1{,}000{,}000$, $\gamma E = \$5{,}000{,}000$, $K = \$1{,}500{,}000$, $r = 5\%$, $R_{\min} = -15\%$, $\alpha = 0.6$ and ρ is uniformly distributed: $\rho \sim U[-20\%, 40\%]$. Note that the utility functions are drawn on different scales (the values on the right hand side are for the CRRA utility function).

7.a



7.b

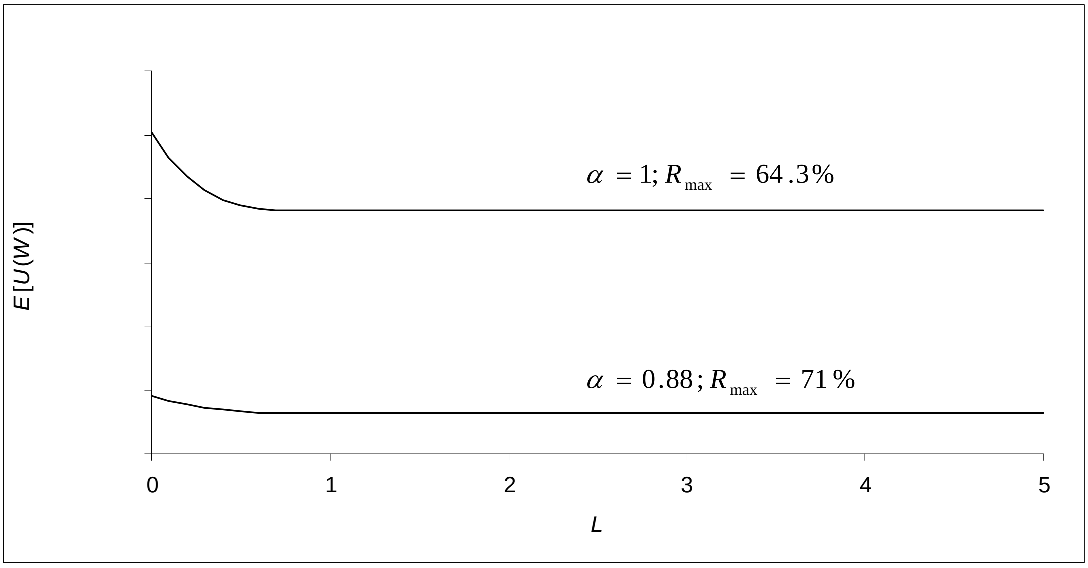


Figure 7. Value function and the agent's optimal leverage policy

Figure 7.a depicts the agent's expected value, $E[V(W)]$, as a function of leverage. The agent's value function is assumed to be either Kahnemen and Trevsky's value function $V(X) = \begin{cases} X^{\alpha} & X > 0 \\ \lambda(-X)^{\alpha} & X \leq 0 \end{cases}$ where $\alpha = 0.88$ and $\lambda = 2.25$ or the bi-linear value function where $\alpha = 1$. Figures 7.b depict the same two cases, only this time $R_{\max}$ is reduced to obtain a leverage-neutral bonus scheme (LNBS). The figures are drawn with the following parameters: $M = \$0$, $W_0 = \$1{,}000{,}000$, $\gamma E = \$5{,}000{,}000$, $K = 800{,}000$, $r = 5\%$, $R_{\min} = -15\%$ and ρ is uniformly distributed: $\rho \sim U[-20\%, 40\%]$

Table 1: CEO's compensation

The table reports the median CEOs' compensation for various industries in the year 2007. The data is taken from Forbes and it composed of four components: Salary (annual base salary earned during the fiscal year), bonus (annual non-equity incentives earned during the fiscal year and discretionary bonuses), other (long-term non-equity incentive payouts, the value realized from vesting of restricted stock and performance shares, executive personal benefits, such as premiums for supplemental life insurance, annual medical examinations, tax preparation and financial counseling fees, club memberships, security services and the use of corporate aircraft), and stock gains (value realized during the fiscal year by exercising vested options granted in previous years).

Industry	Salary	Bonus	Other	Stock Gains
Aerospace & Defense	$1.20 mil	$2.95 mil	$5.30 mil	$5.37 mil
Banking	$0.87 mil	$0.46 mil	$0.63 mil	$0.03 mil
Business Services & Supplies	$0.92 mil	$1.29 mil	$1.10 mil	$0.00 mil
Capital Goods	$1.07 mil	$1.89 mil	$2.26 mil	$10.46 mil
Chemicals	$1.12 mil	$2.11 mil	$0.95 mil	$0.00 mil
Conglomerates	$1.13 mil	$2.60 mil	$2.76 mil	$0.24 mil
Construction	$0.98 mil	$1.10 mil	$1.33 mil	$4.83 mil
Consumer Durables	$1.15 mil	$2.37 mil	$2.92 mil	$0.00 mil
Diversified Financials	$0.85 mil	$2.00 mil	$1.87 mil	$0.00 mil
Drugs & Biotechnology	$1.36 mil	$2.20 mil	$1.13 mil	$3.27 mil
Food, Drink & Tobacco	$1.10 mil	$2.00 mil	$0.84 mil	$0.09 mil
Food Markets	$1.13 mil	$1.95 mil	$1.46 mil	$3.27 mil
Health Care Equipment & Services	$0.98 mil	$1.61 mil	$1.58 mil	$0.00 mil
Hotels, Restaurants & Leisure	$1.18 mil	$2.64 mil	$0.96 mil	$3.02 mil
Household & Personal Products	$1.21 mil	$2.35 mil	$1.96 mil	$0.12 mil
Insurance	$0.98 mil	$1.76 mil	$0.93 mil	$0.00 mil
Materials	$0.98 mil	$1.95 mil	$2.40 mil	$1.38 mil
Media	$1.20 mil	$4.00 mil	$1.20 mil	$0.00 mil
Oil & Gas Operations	$1.08 mil	$1.61 mil	$2.85 mil	$1.51 mil
Retailing	$1.11 mil	$0.62 mil	$0.90 mil	$0.00 mil
Semiconductors	$0.81 mil	$1.40 mil	$0.82 mil	$0.00 mil
Software & Services	$0.71 mil	$0.89 mil	$0.48 mil	$0.00 mil
Technology Hardware & Equipment	$0.95 mil	$1.27 mil	$1.53 mil	$1.60 mil
Telecommunications Services	$1.02 mil	$0.81 mil	$1.89 mil	$0.18 mil
Transportation	$0.60 mil	$0.86 mil	$0.92 mil	$1.09 mil
Utilities	$1.03 mil	$1.33 mil	$2.00 mil	$0.00 mil

Appendix A

In this Appendix, we calculate the derivative of the agent's expected utility with respect to L (Eq. (8)). With this derivative, we show that generally one can obtain a leverage-neutral bonus scheme (LNBS). Differentiating Eq. (6) with respect to L yields:

$$\frac{\partial E[U(W)]}{\partial L} = U(M-K)\frac{\partial}{\partial L}F_\rho(\frac{R_{\min}+rL}{1+L}) + U(M+W_0)[\frac{\partial}{\partial L}F_\rho(\frac{R_T+rL}{1+L}) - \frac{\partial}{\partial L}F_\rho(\frac{R_{\min}+rL}{1+L})] + \frac{\partial}{\partial L}\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} U\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T]\} f_\rho(\rho)d\rho + U[M+W_0+\gamma E(R_{\max}-R_T)]\times\frac{\partial}{\partial L}[1-F_\rho(\frac{R_{\max}+rL}{1+L})]. \tag{A1}$$

To solve the differentiation in Eq. (A1), we use the Liebniz's rule (see, e.g., Kaplan, W. 1992. *Advanced Calculus*. 4th Ed. Reading, Ma: Addison-Wesley, p. 256-258):

$$\frac{\partial}{\partial L}\int_{a(L)}^{b} g(L,\rho)d\rho = \int_{a(L)}^{b(L)} \frac{\partial}{\partial L} g(L,\rho)d\rho + g[L,b(L)]\frac{\partial}{\partial L}b(L) - g[L,a(L)]\frac{\partial}{\partial L}a(L). \tag{A2}$$

Thus, Eq. (A1) can be written as follows:

$$\begin{aligned}\frac{\partial E[U(W)]}{\partial L} &= U(M-K)f_\rho(\frac{R_{\min}+rL}{1+L})(\frac{r-R_{\min}}{(1+L)^2}) + \\ &U(M+W_0)f_\rho(\frac{R_T+rL}{1+L})(\frac{r-R_T}{(1+L)^2}) - U(M+W_0)f_\rho(\frac{R_{\min}+rL}{1+L})(\frac{r-R_{\min}}{(1+L)^2}) + \\ &\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} \frac{\partial}{\partial L}U\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T]\}f_\rho(\rho)d\rho + \\ &U\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T]\}f_\rho(\rho)\Big|_{\rho=\frac{R_{\max}+rL}{1+L}}\times[\frac{\partial}{\partial L}(\frac{R_{\max}+rL}{1+L})] - \\ &U\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T]\}f_\rho(\rho)\Big|_{\rho=\frac{R_T+rL}{1+L}}\times[\frac{\partial}{\partial L}(\frac{R_T+rL}{1+L})] - \\ &U[M+W_0+\gamma E(R_{\max}-R_T)]f_\rho(\frac{R_{\max}+rL}{1+L})(\frac{r-R_{\max}}{(1+L)^2}),\end{aligned} \tag{A3}$$

which can be written as,

$$\frac{\partial E[U(W)]}{\partial L} = U(M-K)\, f_\rho\left(\frac{R_{\min}+rL}{1+L}\right)\left(\frac{r-R_{\min}}{(1+L)^2}\right) +$$
$$U(M+W_0)\, f_\rho\left(\frac{R_T+rL}{1+L}\right)\left(\frac{r-R_T}{(1+L)^2}\right) - U(M+W_0)\, f_\rho\left(\frac{R_{\min}+rL}{1+L}\right)\left(\frac{r-R_{\min}}{(1+L)^2}\right) +$$
$$\gamma E \int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} U'\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T]\}(\rho-r)\, f_\rho(\rho)d\rho + \tag{A4}$$
$$U[M+W_0+\gamma E(R_{\max}-R_T)]\, f_\rho\left(\frac{R_{\max}+rL}{1+L}\right)\left(\frac{r-R_{\max}}{(1+L)^2}\right) -$$
$$U(M+W_0)\, f_\rho\left(\frac{R_T+rL}{1+L}\right)\left(\frac{r-R_T}{(1+L)^2}\right) -$$
$$U[M+W_0+\gamma E(R_{\max}-R_T)]\, f_\rho\left(\frac{R_{\max}+rL}{1+L}\right)\left(\frac{r-R_{\max}}{(1+L)^2}\right).$$

Thus, the second and the sixth expressions and the fifth and the seventh expressions cancel each other out and Eq. (A4) can be written as follows:

$$\frac{\partial E[U(W)]}{\partial L} = [U(M-K) - U(M+W_0)]\, f_\rho\left(\frac{R_{\min}+rL}{1+L}\right)\left(\frac{r-R_{\min}}{(1+L)^2}\right) +$$
$$\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} U'\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T]\}\gamma E(\rho-r)\, f_\rho(\rho)d\rho \equiv P+B. \tag{A5}$$

It is important to note that under reasonable assumptions, which cover most commonly employed utility functions (e.g. linear, CRRA, etc.) or value functions (e.g. bi-linear and K&T Prospect Theory functions), and most common distribution functions (e.g. uniform, normal, lognormal, etc.) both *P*, and *B* in Eq. (5) converge with *L* to zero at the same rate of order 2 (i.e. $B = O(P)$ as $L \to \infty$). This important characteristic guarantees that there is a certain value of $R_{\max}$ for which $P + B \cong 0 \forall L^* \leq L$ where L^* is some threshold leverage. To see this, it is easy to note that $P = O(1/L^2)$ as L^2 appears in the denominator. To see that $B = O(1/L^2)$, note that the two limits of the integral in *B* converge with *L* to *r* at the same rate of order 1. Moreover, because *B* also includes the expression of $(\rho - r)$ and ρ converges to *r* by the limits on the integral, we have a total convergence of order 2. Finally, U' does not change this order of convergence because within U' we have the expression $(\rho - r)L$ such that

$$\lim_{\substack{L\to\infty \\ \rho\to r}} U'\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T] = U'[M+W_0+\gamma E(r-R_T)] > 0 .$$

Appendix B

In this Appendix, we prove Proposition 4. Suppose that the agent considers two policies: L_1 and L_2, where $L_1 < L_2$. Denote by $F(W/L_1)$ and $F(W/L_2)$ the cumulative distributions corresponding to these policies, respectively. Thus, for any preference U, we have:

$$\Delta \equiv E[U(W/L_2)] - E[U(W/L_1)] = \int_{-R_T}^{\infty} [F(W/L_1) - F(W/L_2)]U'(W)dW, \qquad \text{(B1)}$$

(for a proof see Hanoch and Levy, 1969; Hadar and Russel, 1969; and Levy, 2006). As with $R_{\min} < r$, there is only one intersection of $F(W/L_1)$ and $F(W/L_2)$ (see Figure 3), Eq. (B1) can be written as follows:

$$\Delta \equiv A\overline{U}'(A) + B\overline{U}'(B), \qquad \text{(B2)}$$

where $A < 0, B > 0$, and $\overline{U}'(A)$ and $\overline{U}'(B)$ are the *average derivative* of the utility function over the ranges of *A* and *B*, respectively (for a graphical exposition of areas *A* and *B*, see Figure 3). For a risk-neutral preference $\overline{U}'(A) = \overline{U}'(B) \equiv a > 0$. Hence, Eq. (B2) can be rewritten as:

$$\Delta \equiv a(A + B), \qquad \text{(B3)}$$

From Eq. (B3), it emerges that with risk neutrality L_2 dominates L_1 if and only if the area $|A|$ is smaller than the area *B*, a case where $\Delta > 0$. Similarly, L_1 is preferred over L_2 with risk neutrality if $\Delta < 0$, namely $|A| > B$.

Let us now introduce risk aversion and analyze the various segments of *L* given in Figure 6.a, which has been drawn once on the assumption of risk neutrality, where $U(W) = W$, and once on the assumption of a risk averse-preference, where $U(W) = W^{1-\alpha}/(1-\alpha)$. In the segment *ab*, for any two selected values L_1 and L_2 with $L_1 < L_2$, for linear utility function by assumption of the first part of Proposition 4, L_1 provides a higher EU than L_2, hence $\Delta < 0$. In this range L_1 is also preferred over L_2 for risk-averse preferences. The reason for this is that if $\Delta < 0$ for linear preferences, then it must be that $|A| > B$ (see Figure 3 and Eq. (B3)). For risk-averse preferences, Eq. (B2) states that $|A|$ is multiplied by a higher *average marginal* utility than *B*, hence *a fortiori* $\Delta < 0$ also for all risk-averse preferences. Thus, the first part of Proposition 4 has been proven.

Let us consider segment *bc* in Figure 6.a. On this segment, for any L_1 and L_2, where $L_1 < L_2$, we have – according to the assumption the second part of Proposition 4 – that for a linear preference, $\Delta > 0$. Therefore, we must have that $|A| > B$ (see Figure 3). However, with risk preference we have:

$$\Delta \equiv |A|\overline{U}'(A) - B\overline{U}'(B)\,, \tag{B4}$$

and as $\overline{U}'(A)$ is declining over these two segments, we may have that $\Delta < 0$, which completes the proof of the second part of Proposition 4.

Finally, note that for transparency of the proof, we analyze two leverage policies located either on the declining or the increasing segment of the EU, separately (see Figure 6.a). However, the same proof holds true for any two values L_1 and L_2, even if L_1 is located on the declining segment and L_2 is located on the increasing segment.

Thus, we have shown that if L_1 is preferred over L_2 by a risk-neutral preference, $|A| > B$ and *a fortiori* $|A|\overline{U}'(A) > B\overline{U}'(B)$ for risk-averse preferences. If $|A| < B$ (i.e. a higher leverage is preferred by a risk neutral preference) this does not imply that $|A|\overline{U}'(A) < B\overline{U}'(B)$; hence, L_2 may not be preferred by L_1 by a risk-averse preference. Yet, as can be seen from Figure 6.a, risk aversion does not preclude excessive risk taking as EU from a given leverage increases with leverage.

Appendix C

In this Appendix, we calculate the agent's expected utility and expected value given by Eq. (6) for the various utility functions and distributions used in the various graphs of this study. We solve Eq. (6) for linear utility function (Subsection C.1), CRRA utility function (Subsection C.2), and K&T utility function (Subsection C.3). For brevity's sake, the solutions for other utility functions and for other distributions are not given here, as they all reveal the same features.

C.1 Linear utility function (Figures 2, 4, 5, 6)

Substituting $U(W)=W$ in Eq. (6) yields:

$$E[u(W)]=(M-K)F_\rho(\frac{R_{\min}+rL}{1+L})+(M+W_0)[F_\rho(\frac{R_T+rL}{1+L})-F_\rho(\frac{R_{\min}+rL}{1+L})]+$$
$$\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}}\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T]\}f_\rho(\rho)d\rho+[M+W_0+\gamma E(R_{\max}-R_T)]\times$$
$$[1-F_\rho(\frac{R_{\max}+rL}{1+L})]=(M-K)F_\rho(\frac{R_{\min}+rL}{1+L})+ \tag{C1}$$
$$(M+W_0)[F_\rho(\frac{R_T+rL}{1+L})-F_\rho(\frac{R_{\min}+rL}{1+L})]+[M+W_0-\gamma E(rL+R_T)]\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}}f_\rho(\rho)d\rho+$$
$$\gamma E(1+L)\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}}\rho f_\rho(\rho)d\rho+[M+W_0+\gamma E(R_{\max}-R_T)]\times[1-F_\rho(\frac{R_{\max}+rL}{1+L})].$$

Substituting the identity $\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}}f_\rho(\rho)d\rho=F_\rho(\frac{R_{\max}+rL}{1+L})-F_\rho(\frac{R_T+rL}{1+L})$ in Eq. (C1)

yields:

$$E[u(W)]=(M-K)F_\rho(\frac{R_{\min}+rL}{1+L})+$$
$$(M+W_0)[F_\rho(\frac{R_T+rL}{1+L})-F_\rho(\frac{R_{\min}+rL}{1+L})+F_\rho(\frac{R_{\max}+rL}{1+L})-F_\rho(\frac{R_T+rL}{1+L})]-$$
$$\gamma E(rL+R_T)[F_\rho(\frac{R_{\max}+rL}{1+L})-F_\rho(\frac{R_T+rL}{1+L})]+\gamma E(1+L)\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}}\rho f_\rho(\rho)d\rho+ \tag{C2}$$
$$[M+W_0+\gamma E(R_{\max}-R_T)]\times[1-F_\rho(\frac{R_{\max}+rL}{1+L})],$$

which can be written as follows:

$$E[u(W)] = M - KF_\rho(\frac{R_{\min} + rL}{1+L}) + W_0[1 - F_\rho(\frac{R_{\min} + rL}{1+L})] -$$
$$\gamma E(rL + R_T)[F_\rho(\frac{R_{\max} + rL}{1+L}) - F_\rho(\frac{R_T + rL}{1+L})] + \tag{C3}$$
$$\gamma E(R_{\max} - R_T)[1 - F_\rho(\frac{R_{\max} + rL}{1+L})] + \gamma E(1+L)\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} \rho f_\rho(\rho)d\rho,$$

For the uniform distribution, $f_\rho \sim u[a,b]$, (Figure 6) we have that $\int f_\rho(X)dX = F_\rho(X) = (X-a)/(b-a)$ and $\int Xf_\rho(X)dX = (X)^2/2(b-a)$.

For the normal distribution, $f_\rho \sim N[\mu,\sigma]$, (Figures 2, 4, 5) we have that $F_\rho(X) = \Phi[(X-\mu)/\sigma]$, where $\Phi(X)$ is the standard normal cumulative distribution. To solve the integral in Eq. (C3), we add $-\mu$ to the integral and delete the same value:

$$\gamma E(1+L)\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} \rho f_\rho(\rho)d\rho = \gamma E(1+L)[\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} (\rho-\mu) f_\rho(\rho)d\rho + \mu \int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} f_\rho(\rho)d\rho] =$$
$$= \gamma E(1+L)\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} (\rho-\mu) f_\rho(\rho)d\rho + \gamma E(1+L)\mu[\Phi(\frac{\frac{R_{\max}+rL}{1+L} - \mu}{\sigma}) - \Phi(\frac{\frac{R_T+rL}{1+L} - \mu}{\sigma})], \tag{C4}$$

Because $f_\rho(X) = 1/(\sigma\sqrt{2\pi})e^{-(X-\mu)^2/2\sigma^2}$ and correspondingly $\int (X-\mu) f_\rho(X)dX = -(\sigma/\sqrt{2\pi})e^{-(X-\mu)^2/2\sigma^2}$ the integral in Eq. (C4) is given by:

$$\gamma E(1+L)\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} (\rho-\mu) f_\rho(\rho)d\rho = -\gamma E\frac{(1+L)}{\sqrt{2\pi}}\sigma e^{-\frac{(\rho-\mu)^2}{2\sigma^2}}\Bigg|_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} =$$
$$\gamma E\sigma\frac{(1+L)}{\sqrt{2\pi}}(e^{-\frac{(\frac{R_T+rL}{1+L}-\mu)^2}{2\sigma^2}} - e^{-\frac{(\frac{R_{\max}+rL}{1+L}-\mu)^2}{2\sigma^2}}) \tag{C5}$$

Note that by employing a distribution transformation, one can obtain from $f_\rho \sim N[\mu,\sigma]$ that $R_e \sim N((1+L)\mu - rL,(1+L)\sigma)$. Substituting $R_e \sim N((1+L)\mu - rL,(1+L)\sigma)$ in Eq. (3) yields the same solution.

C.2 CRRA utility function (Figure 6)

Substituting $U(W)=\dfrac{W^{1-\alpha}}{1-\alpha}$ in Eq. (6) yields:

$$\begin{aligned}
E[U(W)] &= \frac{(M-K)^{1-\alpha}}{1-\alpha}F_\rho(\frac{R_{\min}+rL}{1+L})+\\
&\frac{(M+W_0)^{1-\alpha}}{1-\alpha}[F_\rho(\frac{R_T+rL}{1+L})-F_\rho(\frac{R_{\min}+rL}{1+L})]+\\
&\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}}\frac{\{M+W_0+\gamma[\rho+(\rho-r)L-R_T]E\}^{1-\alpha}}{1-\alpha}f_\rho(\rho)d\rho+\\
&+\frac{[M+W_0+\gamma(R_{\max}-R_T)E]^{1-\alpha}}{1-\alpha}[1-F_\rho(\frac{R_{\max}+rL}{1+L})].
\end{aligned} \tag{C6}$$

For the uniform distribution, $f_\rho \sim u[a,b]$, we have that $f_\rho(X)=1/(b-a)$ and therefore,

$$\begin{aligned}
&\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}}\frac{\{M+W_0+\gamma[\rho+(\rho-r)L-R_T]E\}^{1-\alpha}}{1-\alpha}f_\rho(\rho)d\rho=\\
&\frac{\{M+W_0+\gamma E[\rho+(\rho-r)L-R_T])\}^{2-\alpha}}{(b-a)(1-\alpha)(2-\alpha)\gamma E(1+L)}\Bigg|_{\frac{R_T+rL}{1+L}}^{\min\left\langle\frac{R_{\max}+rL}{1+L},b\right\rangle}=\\
&\frac{[M+W_0+\gamma E(\min\langle R_{\max},b(1+L)-rL\rangle-R_T)]^{2-\alpha}-(M+W_0)^{2-\alpha}}{(b-a)(1-\alpha)(2-\alpha)\gamma E(1+L)}.
\end{aligned} \tag{C7}$$

C.3 K&T utility function (Figure 7)

Substituting K&T value function, $U(X)=\begin{cases}X^\alpha & X>0\\ -\lambda(-X)^\alpha & X\le 0\end{cases}$, in Eq. (6),

where *X* is in terms of the change in wealth (i.e. $M=0$) yields:

$$\begin{aligned}
E[U(W)] &= -\lambda(K)^\alpha F_\rho(\frac{R_{\min}+rL}{1+L})+(W_0)^\alpha[F_\rho(\frac{R_T+rL}{1+L})-F_\rho(\frac{R_{\min}+rL}{1+L})]+\\
&\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}}\{W_0+\gamma E[\rho+(\rho-r)L-R_T]\}^\alpha f_\rho(\rho)d\rho+\\
&+[W_0+\gamma E(R_{\max}-R_T)]^\alpha\times[1-F_\rho(\frac{R_{\max}+rL}{1+L})].
\end{aligned} \tag{C8}$$

For the uniform distribution, $f_\rho \sim u[a,b]$, (Figures 6) we have that

$f_\rho(X)=1/(b-a)$ and therefore,

$$\int_{\frac{R_T+rL}{1+L}}^{\frac{R_{\max}+rL}{1+L}} \{W_0+\gamma[\rho+(\rho-r)L-R_T]E\}^{\alpha} f_{\rho}(\rho)d\rho =$$

$$\left.\frac{\{W_0+\gamma E[\rho+(\rho-r)L-R_T])\}^{1+\alpha}}{(b-a)(1+\alpha)\gamma E(1+L)}\right|_{\frac{R_T+rL}{1+L}}^{\min\left\langle\frac{R_{\max}+rL}{1+L},b\right\rangle} = \tag{C9}$$

$$\frac{[W_0+\gamma E[\min\langle R_{\max},b(1+L)-rL\rangle-R_T)]^{1+\alpha}-(W_0)^{1+\alpha}}{(b-a)(1+\alpha)\gamma E(1+L)}.$$