Governance through Threats of Intervention and Exit[☆]

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Abstract

An activist shareholder can discipline management through the threat of intervention and threat of exit. Intervention occurs in equilibrium when the activist has a toehold almost sufficient for exercising voice, the activist is effective in restoring firm value, or the temptation for misbehavior by the management is large. Corporate governance is most effective when activists rarely act: When the threat of intervention plays a strong disciplinary role there are fewer ex-post interventions; when the threat of exit plays a strong disciplinary role there are fewer block sales. The activist's choice of size of toehold effectively determines equilibrium form of governance.

One of the fundamental issues in modern corporate finance is the problem of separation of firm ownership from control. The gap between management and shareholders is potentially wide and the danger is great for agency problems to divert a widely-held firm's resources from their efficient use. Therefore it is important to understand what mechanisms are available for reconciling these interests, to what extent they are used, and to what extent they are effective.

If a shareholder decides he does not like what a firm's management is doing, he has two alternatives: He can *intervene* or he can *exit*—that is, he can work on changing the firm's manager's behavior directly or he can sell his shares. Intervention, sometimes referred to as "voice," includes a variety of possible actions to compel changes in behavior by management: replacement of boards of directors, support for takeover bids, and proxy initiatives to limit management discretion or to affect management compensation. Exit, sometimes referred to as the "Wall Street walk," is the sale of shares in a firm when the shareholder disapproves of management actions but does not choose to engage in direct intervention.

However, both exit and intervention can also have indirect effects, because the foreknowledge by managers of the possible reactions of dissatisfied shareholders can alter managerial behavior. Thus we are not only interested in exit and intervention as behaviors by the blockholder, we are also interested in how they affect managerial behavior. That is, we are also interested in the incentive effects on managers of the *threats* of shareholder exit or intervention.

Beginning with Admati and Pfleiderer (2009) and Edmans (2009), a series of recent articles have shown that, provided management compensation is tied in the short run to share price, the threat of exit and the resultant reduction of share price, can serve as a disciplinary device. Despite empirical investigations of each of these strands of governance, surprisingly little theoretical attention has been paid to the question of comparing the factors that lead to the adoption of exit or of intervention. Moreover, no theoretical attention has been paid to the question of how *the threat of intervention* affects the choice of these two governance mechanisms in equilibrium.

In this paper we ask the following research questions. What type of shareholder action—intervention or exit—can exist as an equilibrium response to managerial misbehavior? Under what circumstances can intervention and exit play stronger disciplinary roles? Do more frequent ex post interventions mean that corporate governance is functioning well? Under what circumstances will an activist shareholder choose a form of governance that leads to the highest firm value as an equilibrium response to managerial misbehavior?

To address these important questions, we provide a simple model in which an activist shareholder can accumulate a toehold and then discipline management through the threat of exit and threat of intervention. Once the toehold is established, the activist can decide to sell shares on observing a negative signal (i.e., exit). Alternatively, the activist can decide to extend the toehold and intervene when there is a potential for value creation. In anticipation of these actions, the management is more restrained in consumption of private benefits.

In the model, an important role is played by the market's revelation through prices of the activist's response to managerial behavior. If market prices are unaffected, there is no channel through which a blockholder's actions can influence the manager in the exit equilibrium. On the other hand, if the market fully reveals the activist's private information, the activist has no incentive to sell rather than hold on to the assets. Moreover, when prices are fully revealing the activist has no incentive to accumulate the toehold in the first place. For this reason it is important to consider the effect of liquidity trading on the mechanisms; the presence of liquidity trades enables the activist to a certain degree to hide his information. We focus in this paper on liquidity trades by the activist himself, since these are the most effective sources of information clouding.

Our research framework is relevant in modern financial markets, because most publicly traded firms can be subject to either type of governance. Because the effects of these mechanisms on managerial behavior are often unobservable to empiricists, theoretical analysis of what form of governance disciplines the manager can enhance our understanding of how financial markets operate. We therefore investigate the circumstances that encourage the use of one of the mechanisms rather than the other in equilibrium. We also consider the cases where two equilibria exist, one with each mechanism, and consider the factors that lead to the greater effectiveness of one or the other mechanism.

The model reveals several key results. First, we ask what type of shareholder action—intervention or exit—can exist as an equilibrium response to managerial misbehavior. We find that the intervention equilibrium is more likely to exist when the costs of intervention are low – in particular, when the activists builds an initial toehold almost big enough to be an effective stake for exercising voice – or when the activist is particularly effective in restoring firm value. Moreover, increases in the temptation for misbehavior by the management increase the chances for an intervention equilibrium. In all of these cases the effect on existence of an exit equilibrium is reversed. Thus, variations in the size of the initial toehold, the activist's effectiveness in restoring firm value, and the temptation for misbehavior by the management generate a *substitution* between two governance mechanisms.[1]

Second, when intervention is the equilibrium form of governance, disciplinary pressure on the manager is greater as the activist has greater power to penalize deviating managers and has smaller power to restore firm value. The effect of liquidity shocks that force him to sell the block on manager's incentives can be either positive or negative. For example, when the Intervention equilibrium is disciplinary, smaller chances of a liquidity shock lead to narrower bid-ask spreads (i.e., better measured liquidity) and to stronger disciplinary pressure on the manager. Importantly, the stronger disciplinary role played by the intervention mechanism leads to *fewer* ex post interventions in equilibrium. In the extreme, one would not observe any intervention events if the threat of intervention were so powerful as to prevent the manager from taking the bad action in any state of the world.[2] Thus, when the in-

[1]In addition, we establish conditions in which the lone equilibrium outcome is a mixed strategy, including probabilities of both exit and intervention. This result is reminiscent of several papers which have shown in more complicated contexts that exit and voice can be complementary (e.g., Dasgupta and Piacentino, 2014); in our model it can be thought of as occurring in cases where neither mechanism is strong enough to survive on its own.

[2]In this extreme case the situation bears a similarity to the theory of contestible markets, where potential competition, even though unobserved, manages to provide market discipline against temptations toward inefficient behavior (Baumol et al., 1988).

tervention equilibrium is disciplinary, conditional on activist having a block, higher stock liquidity is associated with fewer interventions and with more discipline imposed on the manager through the threat of intervention. This prediction finds support in Edmans et al. (2013), who show that conditional on having a large block, shareholders are less likely to intervene when measured liquidity increases. Similarly, Back et al. (2013) find that higher stock liquidity leads to fewer interventions by large blockholders. Whereas these papers conclude that higher measured liquidity is harmful for the "voice" form of governance, our model shows that it actually facilitates governance through the threat of intervention.

Third, we analyze the robustness of governance mechanisms to changes in the structure of the manager's compensation. We find that if the manager's compensation were more aligned with the long-term firm value, the disciplinary role played by the intervention (exit) mechanism would increase (decrease). This result suggests that exit becomes more preferred as a form of governance the stronger the exogenous (i.e., un-modeled) factors that lead to short-termism.

Fourth, several interesting findings come from the analysis of liquidity shocks. Typically, bid-ask spreads are used empirically as a measure of stock liquidity. We find that in the exit equilibrium firm value is lower and the bid-ask spread is narrower when the activist is more likely to face sell-side liquidity shocks. Thus, the variation in the liquidity shock implies a negative association between measured stock liquidity and firm value. When we study the role of liquidity shocks in a disciplinary intervention equilibrium, we find that firm value is lower and the bid-ask spread is wider when the activist is more likely to face liquidity shocks. Overall, the results suggest that the type of the equilibrium form of governance plays key role in determining the relation between endogenously determined stock liquidity and firm value.

In the final section of the paper, we endogenize the activist's choice of initial toehold. The size of the toehold is increasing in stock market liquidity and decreasing in the cost associated with holding the toehold. We conclude the analysis by asking under what conditions the activist's private choice leads to the most effective form of governance (i.e., one that leads to the highest firm value). We show that there can be a discrepancy between the activist's private choice and the effective outcome, so that, for example, when the exit form of governance is more effective, conditions which encourage amassing a large initial toehold actually decrease the chances of introducing the effective form of governance.

Related Literature

This paper is related to several strands of the corporate governance literature that studies the role of blockholders in reducing agency costs.[3]

First, the paper contributes to the strand of literature that studies how shareholder intervention can increase firm value ex post (e.g., Shleifer and Vishny, 1986; Kyle and Vila, 1991; Admati et al., 1994; Maug, 1998; Bolton and von Thadden, 1998; Kahn and Winton, 1998; Noe, 2002; Faure-Grimaud and Gromb, 2004; Collin-Dufresne and Fos, 2014). For example, in their classic paper Shleifer and Vishny (1986) show that the presence of a large minority shareholder provides a partial solution to the free-rider

[3]Edmans (2013) surveys theoretical and empirical literature on the role of blockholders in corporate governance.

problem and therefore reduces the agency costs. In this strand of literature, intervention does not play a disciplinary role. Intervention occurs in the absence of managerial action; more effective monitoring does not change the manager's incentives and therefore is beneficial for shareholders only because it increases firm value *ex post*.

Second, the paper contributes to the literature that studies how corporate governance can affect management's incentives. Grossman and Hart (1980) were the first to argue that managers face trade offs between a high profit action with an associated low chance of being raided and a low profit (but high managerial-utility) action which leads to a successful takeover bid. In their model managers are more reluctant to take self-serving actions that lower firm value and increase the probability of a takeover. Scharfstein (1988) explicitly models the source of contractual inefficiencies which was not studied by Grossman and Hart (1980). He explores the conditions under which the takeover threat plays a genuine role (beyond incentive contracts) in disciplining management.[4] The literature has also studied the governance role of exit and showed that a large shareholder can alleviate conflicts of interest between managers and shareholders through the credible threat of exit on the basis of private information (e.g., Admati and Pfleiderer, 2009; Edmans, 2009). Within this group, our paper is most closely related to Admati and Pfleiderer (2009) whose modeling setup we adopt for the exit form of

[4]While the above papers show that takeover plays a positive disciplinary role, several other papers have highlighted some negative aspects of the threat of intervention (e.g., Stein, 1988; Zwiebel, 1996; Burkart et al., 1997). For example, Stein (1988) develops a model in which takeover pressure can be damaging because it leads managers to sacrifice long-term interests in order to boost current profit.

governance.

Our paper contributes to the corporate governance/management incentive literature not only by jointly considering the effect of two main corporate governance mechanisms in improving management's incentives, but also by studying how the threat of intervention affects the choice of these two governance mechanisms in equilibrium. Very few papers consider the disciplinary roles of both exit and the intervention mechanisms in resolving an agency conflict between shareholders and management. Dasgupta and Piacentino (2014) show that when money managers compete for investor capital, the threat of exit loses credibility, weakening its governance role. When they allow funds to engage in activist measures, they find that the threat of exit and intervention are complementary in generating good governance, because blockholders will use intervention if and only if they can credibly threaten to exit.[5] Levit (2012) interprets voice as a strategic transmission of information from an activist investor to an opportunistic manager. He shows that this type of voice and exit exhibit complementarity. Edmans and Manso (2011) also investigates a structure in which both trade and direct action are available to blockholders. In their model the manager does not expend enough

[5]Our model differs from Dasgupta and Piacentino (2014) on several key dimensions. First, in Dasgupta and Piacentino (2014) the blockholder does not have to choose between two mechanisms. Instead, the blockholder can engage in exit *after* an intervention attempt. In our model, the activist faces a choice between the two, generally substitute, mechanisms. Second, while in our model the activist can improve firm value and impose a private cost on the manager if the bad action is taken, in Dasgupta and Piacentino (2014) he can only impose a private cost on the manager. Our modeling assumption is consistent with robust empirical evidence on value creation by activist shareholders (e.g., Brav et al., 2008). Finally, in Dasgupta and Piacentino (2014) the activist owns a sufficient number of shares and therefore does not need to trade in order to become active. In our model the activist is required to trade in order to become active.

effort because prices do not fully reflect the impact of manager's action on firm value. They focus on the role of multiple small blockholders and show that while such a structure generates free-rider problems that hinder intervention, the same coordination difficulties strengthen a second governance mechanism: disciplining the manager through trading.

1. Setup

In the basic model there are three periods 0, 1, and 2 and three types of agents: the manager, whom we denote by $\mathcal{M}$, an activist shareholder, $\mathcal{A}$, who owns φ shares in the firm, and a continuum of uninformed traders (the "market makers"). Markets for shares in the firm occur in periods one and two. In Section 4 we analyze a period prior to period 0 in which $\mathcal{A}$'s choice of initial holding φ is endogenized. The choice of optimal φ will take into account the impact of φ on $\mathcal{M}$'s incentives (and therefore firm value and $\mathcal{A}$'s profits) and the cost of holding φ shares (e.g., lack of diversification, effort spend on gathering information).

In period 0, $\mathcal{M}$ decides whether or not to take a particular action. An agency problem arises because $\mathcal{M}$ and the shareholders have conflicting preferences with respect to the action. Specifically, we assume the action is "bad" in the sense that it reduces the value of the firm, but provides a private benefit to $\mathcal{M}$. The benefit has the positive value β, known with certainty by all participants.[6] The cost of the damage to the firm is $\tilde{\delta}$, a random value

[6] Fos and Jiang (2015) document evidence consistent with a manager's value of private benefits of control being 5%-20% of the stock price when the company is targeted in proxy contest.

which $\mathcal{M}$ learns privately immediately before making his decision.[7] Let the decision be denoted a (either zero or one); then the value of the firm in period 2 will be $\upsilon - a\tilde{\delta}$, in the absence of intervention by the activist. The value υ is common knowledge. All agents know that the value $\tilde{\delta}$ is drawn from a continuous distribution $F(.)$ with density $f(.)$ and support $[0, \bar{\delta}]$, where $\bar{\delta}$ is sufficiently large. When illustrate some results we will further assume that the distribution of δ is exponential with $F(\delta) = 1 - e^{-\delta\lambda}$.

$\mathcal{M}$'s strategy can be described by defining the set $\Delta \subseteq [0, \bar{\delta}]$, such that $a = 1$ if and only if δ is in the set Δ. Let $\Phi = Pr\{\delta \in \Delta\}$, the ex ante probability that $\mathcal{M}$ chooses $a = 1$.

In period 1, $\mathcal{A}$ observes the action taken by $\mathcal{M}$. Given $\mathcal{M}$'s strategy, observing $\mathcal{M}$'s actions provides $\mathcal{A}$ with a noisy signal of $\tilde{\delta}$. Then $\mathcal{A}$ must decide whether to buy, sell, or hold his shares at the period 1 market. If $\mathcal{A}$ buys sufficient shares, he can *intervene* in period 2, reducing the benefit to $\mathcal{M}$ of taking the bad action, and reducing the damage of the action to the firm. Specifically, if $\mathcal{A}$ intervenes then the benefit to $\mathcal{M}$ is reduced to $\beta\gamma$ and the value of the firm is restored to $\upsilon - a\tilde{\delta}\kappa$, where $0 < (1-\gamma) < 1$ measures the effectiveness of $\mathcal{A}$ in reducing the private benefits of control and $0 < (1-\kappa) < 1$ measures his effectiveness in restoring firm value. Let $b \in \{0, 1\}$ represent the decision to intervene. Then the ultimate value of the firm is $\upsilon - a\tilde{\delta}(\kappa b + 1 - b)$. We assume this value is publicly revealed before the market at the end of period 2, so that trade in the final market occurs

[7]In a supplement to this paper we also consider the case where $\mathcal{M}$'s action is "good" in that it increases the firm's value at a private cost to the manager. For the most part that G version of the model (to use the terminology of Admati and Pfleiderer) provides results parallel to the B version adopted here.

at this price.[8]

As we are going to see later on, both $\mathcal{A}$'s ability to reduce the benefit to $\mathcal{M}$ of taking the bad action, $(1-\gamma)$, and $\mathcal{A}$'s ability to reduce the damage of the action to the firm, $(1-\kappa)$, will play an important role in the model. Whereas $(1-\kappa)$ will be one of key parameters to determine what governance mechanism exists in equilibrium (intervention or exit), both $(1-\gamma)$ and $(1-\kappa)$ will determine the degree of discipline imposed on $\mathcal{M}$ through the threat of intervention. Consistently with $(1-\gamma) > 0$, Fos and Tsoutsoura (2014) show that activist shareholders are able to impose a significant career cost on directors of targeted companies. Directors of companies that experience a proxy contest lose seats not only on boards of targeted companies, but also on boards of other companies. Several pieces of evidence motivate $(1-\kappa) > 0$. For example, Brav et al. (2008) show that firm value increases upon intervention by activist hedge funds. Similarly, Fos (2015) shows that firm value increases upon announcements of a proxy contest, which is probably the most hostile type of shareholder activism.

We next characterize $\mathcal{A}$'s trading in period 1. If $\mathcal{A}$ decides to sell shares after observing $\mathcal{M}$'s action, he sells the entire position φ. As we are going to see later on, in any equilibrium in which $\mathcal{A}$ decides to sell shares, it is optimal to sell the entire position. If $\mathcal{A}$ decides to buy shares, he buys $\alpha-\varphi$

[8] In our formulation, the power of the blockholder to punish managers or to repair damage to the firm are simply taken as parametric. One formulation which has endogenized the ability of outsiders to punish management is that of Fluck (1999). In her account, the size of outsider holdings affects the likelihood of being able to remove the manager. Since she assumes that the firm is less valuable when managers are removed from control, such a threat is not credible except in an infinitely repeated game. She examines credible threats in that framework.

shares. Both α and φ are publicly known in period 1. $\mathcal{A}$'s choice of φ is endogenized in Section 4. Empirically, φ could correspond to stockholder's ownership disclosed in Schedule 13F filings.

Several factors could affect α. For example, a larger α could correspond to cases when $\mathcal{A}$ needs more voting power to make the intervention effective. A larger α could also correspond to a higher direct cost of intervention. Instead of modelling one specific channel that determines α, we leave it as an exogenous parameters in the model. The difference between α and φ therefore captures the degree of $\mathcal{A}$'s dependence on financial markets. High $(\alpha - \varphi)$ means that $\mathcal{A}$ would need to purchase many shares in order to intervene. Interventions by activist shareholders are a good example of this scenario because activist shareholders often start accumulating shares when they own about 3-4% of outstanding shares (φ) and then they end up with about 7% of outstanding shares (α) (Brav et al., 2008; Collin-Dufresne and Fos, 2015). Small $(\alpha - \varphi)$ means that $\mathcal{A}$ would need to purchase only a few shares in order to intervene. Activism campaigns aimed at passing shareholder proposals sponsored by pension funds are a good example of this scenario because such campaigns involve little (if any) trading by activists.[9]

$\mathcal{A}$'s trades in period 1 may reveal information both about $\mathcal{M}$'s actions and about $\mathcal{A}$'s own intentions for period 2. We will assume that this

[9]We have also considered a case when there is a direct cost of intervention. In that case, all solutions become more complicated, because prices reflect not only the probability that the bad action is taken, but also the probability the damage to firm value is large enough. When $\mathcal{M}$ knows that the damage of his action to firm value is large enough to justify the intervention, there is no material change in the results. However, when $\mathcal{M}$'s action implies a small damage to firm value, $\mathcal{M}$ realizes that $\mathcal{A}$ won't intervene because the intervention will not recover enough damage to make the intervention profitable. Intervention plays no disciplinary role in this case. Results are available upon request.

information will be somewhat obscured by additional liquidity needs of $\mathcal{A}$. Specifically, we assume that $\mathcal{A}$ in period 1 will with probability θ suffer a liquidity shock which requires him to divest himself of any holdings of firm shares and which prevents him from purchasing any shares of the firm. If he does not suffer a liquidity shock, then his purchases and sales will be based on his information and his strategy for future intervention. Other participants in the market are unable to observe the liquidity shock of $\mathcal{A}$, and so the price prevailing will take into account their expectation of the relative likelihood of the shock.

$\mathcal{M}$'s compensation is assumed to be linear in the realized market price of the firm in periods 1 and 2, P_1 and P_2. Specifically, we assume that compensation is equal to $\omega_1 P_1 + \omega_2 P_2$, where ω_1 and ω_2 are positive coefficients representing the dependence of the compensation on the firm's short-term ("Period 1") and long-term ("Period 2") price performance, respectively.[10] $\mathcal{M}$ chooses whether to take the action or not to maximize his expected utility for every realization of $\tilde{\tilde{\delta}}$.

When $\mathcal{A}$ is not present, $\mathcal{M}$'s preferred cutoff point, denoted δ_{BM}, is equal to β/ω_2. That is, $\mathcal{M}$ takes the action when $\widetilde{\delta} \leq \delta_{BM} = \beta/\omega_2$. $\mathcal{A}$'s role in governance will be measured by his impact on firm value. Let p_{BM} denote the expected value of the firm when $\mathcal{A}$ is not present.

Next consider the case when $\mathcal{A}$ is present. If $\mathcal{M}$ does not take the

[10] $\mathcal{M}$'s sensitivity to short-term prices is taken as exogenous in this paper. It can be motivated, for example, by takeover threats and concern for managerial reputation (Edmans, 2009). Neither existence nor the disciplinary role of the Intervention equilibrium is affected if we set $\omega_1 = 0$. However, whereas the Exit equilibrium can exist when $\omega_1 = 0$, it plays no disciplinary role.

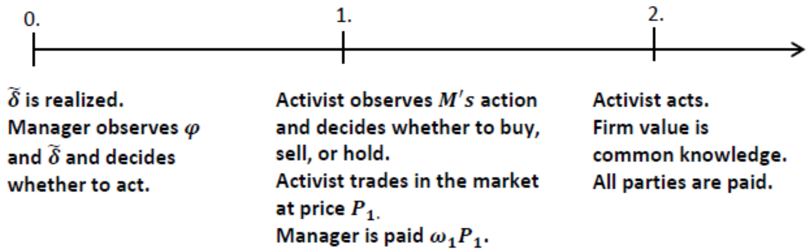


Figure 1: **Time-line.**

action, then $\mathcal{M}$'s utility is simply his compensation, $\omega_1 P_1 + \omega_2 P_2$. If he takes the action, then his utility depends on $\mathcal{A}$'s period 1 trading decision and on any resultant intervention in period 2. If intervention does not occur, $\mathcal{M}$'s utility is equal to the sum of his compensation and the private benefit β. If intervention occurs, $\mathcal{M}$'s utility is equal to the sum of his compensation and the private benefit $\beta\gamma$. Note that in both cases prices will reflect $\mathcal{A}$'s period 1 trading decision and the decision to intervene in period 2.

When the intervention mechanism operates, the potential impact of $\mathcal{A}$ on $\mathcal{M}$'s decision comes about through the impact of his trading decisions on P_1, through his impact on firm value at period 2, and through his impact on private benefits of control. When the exit mechanism operates, the potential impact of $\mathcal{A}$ on $\mathcal{M}$'s decision comes about only through the impact of his trading decisions on P_1. We assume that prices are set by risk-neutral, competitive market makers and therefore reflect all of the information publicly available. This means, as noted before, that P_2 equals $\upsilon - a\delta(\kappa b + 1 - b)$. In period 1, P_1 reflects the information contained in $\mathcal{A}$'s trading decision. The timing of events is given in Figure 1.

2. Solving the Model

We assume $\mathcal{A}$ is restricted to three actions $T \in \{B, H, S\}$ in period 1: buy enough to get the level to the required amount for intervention (B); sell all holdings (S); or keep holdings unchanged (H for "hold").

It is useful to introduce notation for the prices that would occur if uninformed agents observed $\mathcal{M}$'s action (denote these as p_T^a). If $a = 0$, $p_T^0 = \upsilon$ for any T. If $a = 1$, $p_H^1 = p_S^1 = \upsilon - \Lambda$, where $\Phi \equiv Pr(\delta \in \Delta)$ is the probability of $\mathcal{M}$ taking the action and $\Lambda \equiv \Phi^{-1} E[\mathbb{1}_{\delta \in \Delta} \delta]$ is the expected damage to firm value, conditional on $\mathcal{M}$ taking the action. Note that the price if held is the same as the price if sold, because without having enough of a holding to intervene, $\mathcal{A}$ adds no value to the asset. Finally, $p_B^1 = \upsilon - \kappa\Lambda > p_H^1 = p_S^1$, reflecting the benefit from intervention.[11]

We next consider the value of $\mathcal{A}$'s position. The value from standing pat is $\pi_0^H = \varphi\upsilon$ if $a = 0$ and $\pi_1^H = \varphi(\upsilon - \Lambda)$ if $a = 1$. The value from selling the lot is $\pi_a^S = \varphi p_S$ (note this does not actually depend on a). The value from buying is $\pi_0^B = \alpha\upsilon - (\alpha - \varphi) p_B$ if $a = 0$ and $\pi_1^B = \alpha(\upsilon - \kappa\Lambda) - (\alpha - \varphi) p_B$ if $a = 1$. Hereafter, we will refer to the value *net* of the initial holding $\varphi\upsilon$ as "$\mathcal{A}$'s profits."

A *market equilibrium* for period 1 specifies the probability mixture for $\mathcal{A}$ between buy, hold, and sell $\left(\sigma_a^B, \sigma_a^H, \sigma_a^S\right)$, for $a = 1$ or 0, *conditional on no liquidity shock* and market prices p_B, p_S, such that the probabilities are max-

[11] We know that $\Phi > 0$ (and so $\Lambda > 0$) because for any fixed values of ω_1 and ω_2, for δ sufficiently close to zero, $\mathcal{M}$ would prefer to take the action, even if it were publicly observable, and therefore reduced prices in both periods 1 and 2.

imizing choices given prices, and prices are consistent with the probabilities.

$$\sum_{T=B,H,S} \sigma_a^T \pi_a^T \geq \pi_a^{T'} \text{for all } T' \in \{B,H,S\}, \text{ for } a = 0,1.$$

$$p_T^1 \leq p_T \leq p_T^0, \text{ for } T \in B,S$$

$$\begin{aligned} p_S &= \frac{(1-\theta)[p_S^1 \Phi \sigma_1^S + p_S^0 (1-\Phi)\sigma_0^S] + \theta[p_S^1 \Phi + p_S^0(1-\Phi)]}{(1-\theta)[\Phi\sigma_1^S + (1-\Phi)\sigma_0^S] + \theta} \\ &= \upsilon - \Lambda \frac{\Phi\sigma_1^S + \bar{\theta}\Phi}{\Phi\sigma_1^S + (1-\Phi)\sigma_0^S + \bar{\theta}} \\ p_B &= \frac{(1-\theta)[p_B^1 \Phi \sigma_1^B + p_B^0(1-\Phi)\sigma_0^B]}{(1-\theta)[\Phi\sigma_1^B + (1-\Phi)\sigma_0^B]} = \upsilon - \kappa\Lambda \frac{\Phi\sigma_1^B}{\Phi\sigma_1^B + (1-\Phi)\sigma_0^B}, \end{aligned}$$

where $\bar{\theta} \equiv \theta/(1-\theta)$ and p_B is defined when the denominator is non-zero. Without loss of generality we can specify p_B when the denominator is zero. If there is zero probability of buying, we can set $p_B = p_B^0$. To see this, note that if $p_B < p_B^0$ in an equilibrium with no buying, then higher buying prices also yield an equilibrium with the same allocation; moreover p_B cannot exceed p_B^0 in equilibrium. The following Lemma shows that we can put more structure on equilibrium beliefs.

Lemma 1. *In equilibrium,* $\sigma_0^S = 0$ *and* $\sigma_1^H = 0$.[12]

$\mathcal{A}$'s profits, as measured by $\pi - \varphi\upsilon$, are presented in Table 1.

2.1. Equilibrium with Intervention

We begin by characterizing equilibria in which $\mathcal{A}$ intervenes with a positive probability, i.e., when $\sigma_1^B > 0$. Let $\Phi_I, \Lambda_I, \delta_I$ denote the equilibrium

[12]All proofs are in the appendix.

Table 1: **$\mathcal{A}$'s Profits.** This table describes profits of $\mathcal{A}$, as measured by $\pi - \varphi \upsilon$, adjusted to results of Lemma 1. The left column reports profits if $\mathcal{M}$ does not take the action and the right column reports profits if $\mathcal{M}$ takes the action. In the case of each bracketed expression, the second term is to be used in case the denominator of the first term is zero.

	$a = 0$ (no damage)	$a = 1$ (damage)
Buy	$(\alpha - \varphi)\kappa\Lambda\{\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B + (1-\Phi)\sigma_0^B}, 0\}$	$-\alpha\kappa\Lambda + (\alpha - \varphi)\kappa\Lambda\{\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B + (1-\Phi)\sigma_0^B}, 0\}$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda\frac{\Phi(1-\sigma_1^B)+\bar{\theta}\Phi}{\Phi(1-\sigma_1^B)+\bar{\theta}}$	$-\varphi\Lambda\frac{\Phi(1-\sigma_1^B)+\bar{\theta}\Phi}{\Phi(1-\sigma_1^B)+\bar{\theta}}$

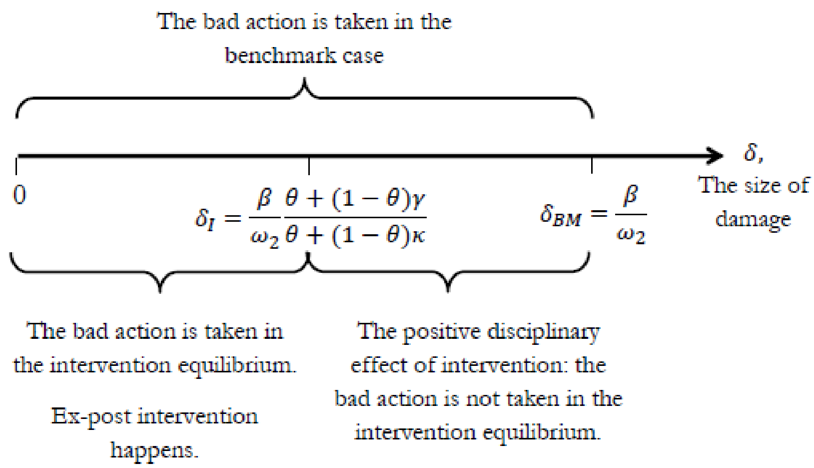
values of Φ, Λ, δ.

Proposition 1. *There exists a unique equilibrium with $\sigma_1^B = 1$ if and only if*

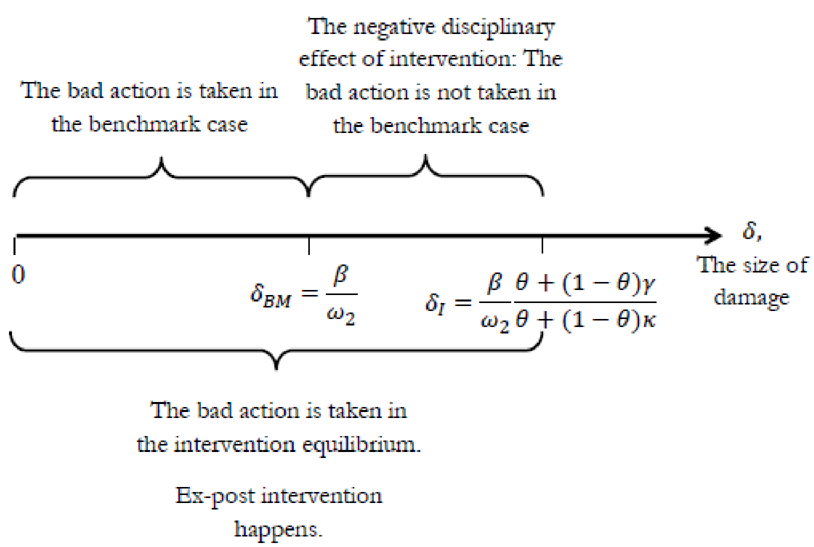
$$\frac{\varphi}{\alpha} > \frac{\kappa}{(1-\kappa)} \frac{(1-\Phi_I)}{\Phi_I}, \quad (1)$$

where $\delta_I = \frac{\beta}{\omega_2} \frac{\theta + (1-\theta)\gamma}{\theta + (1-\theta)\kappa}$, $\Phi_I = F(\delta_I)$, and $\Lambda_I = \Phi_I^{-1} E[\mathbb{1}_{\delta<\delta_I}\delta]$. Equilibrium beliefs are $\left(\sigma_0^B = 1, \sigma_0^H = 0, \sigma_0^S = 0; \sigma_1^B = 1, \sigma_1^H = 0, \sigma_1^S = 0\right)$. Equilibrium prices are $p_B = \upsilon - \kappa\Lambda_I\Phi_I$ and $p_S = \upsilon - \Lambda_I\Phi_I$. The equilibrium is disciplinary if and only if $\gamma < \kappa$.

The impact of the intervention on $\mathcal{M}$'s incentives is illustrated in Figure 2. Whether the impact is positive or negative depends on the relation between γ and κ. When $\gamma < \kappa$ ($\mathcal{A}$ is more effective in reducing the private benefits than in restoring firm value), the intervention plays a *positive* disciplinary role ($\delta_I < \delta_{BM}$). In contrast, when $\gamma > \kappa$ ($\mathcal{A}$ is more effective in restoring firm value than in reducing the private benefits), the intervention plays a *negative* disciplinary role ($\delta_{BM} < \delta_I$)—that is, the manager chooses the bad action more frequently than he would if $\mathcal{A}$ were absent.



(a) $\gamma < \kappa$: Positive disciplinary role of Intervention.



(b) $\gamma > \kappa$: Negative disciplinary role of Intervention.

Figure 2: The disciplinary role of Intervention.

Panel A in Figure 2 depicts the case when $\gamma < \kappa$. When $\delta > \delta_{BM}$, $\mathcal{M}$ does not take the bad action even when $\mathcal{A}$ is not present. In this case the damage to firm value is so large that $\mathcal{M}$ prefers to forego the private benefit β. In the intermediate region $\delta_{BM} > \delta > \delta_I$, $\mathcal{A}$'s presence prevents $\mathcal{M}$ from taking the bad action. This is the disciplinary role of the intervention. Finally, when $\delta < \delta_I$, $\mathcal{M}$ takes the bad action and ex post intervention takes place. Only in this region will market participants observe incidents of intervention in the case $\gamma < \kappa$.

Panel B in Figure 2 depicts the case when $\gamma > \kappa$. When $\delta > \delta_I$, $\mathcal{M}$ does not take the bad action even when $\mathcal{A}$ is present. When $\delta < \delta_I$, $\mathcal{M}$ takes the bad action and ex post intervention takes place. Notice that $\mathcal{M}$'s bad action is induced by the presence of $\mathcal{A}$ when $\delta_{BM} < \delta < \delta_I$.

Overall, how does $\mathcal{A}$'s effectiveness at restoring firm value $(1-\kappa)$ impact equilibrium firm value? On one side, a higher $(1-\kappa)$ increases $\mathcal{M}$'s incentive to take the bad action because the damage to firm value will be partially recovered. This is the negative impact of $(1-\kappa)$ on firm value. On the other side, higher $(1-\kappa)$ means that ex post intervention will create more value. The overall effect therefore depends on the strength of these two effects. Figure 3 shows that the response is not monotonic in κ. When $\mathcal{A}$'s effectiveness in restoring firm value is high ($(1-\kappa)$ is high), the positive effect that operates through ex post value creation dominates the negative effect that operates through $\mathcal{M}$'s incentives. In this case, a *more* productive $\mathcal{A}$ achieves a higher firm value. In contrast, when $\mathcal{A}$'s effectiveness in restoring firm value is small ($(1-\kappa)$ is low), the negative effect dominates the positive effect, and a *less* productive $\mathcal{A}$ achieves higher firm value. Note,

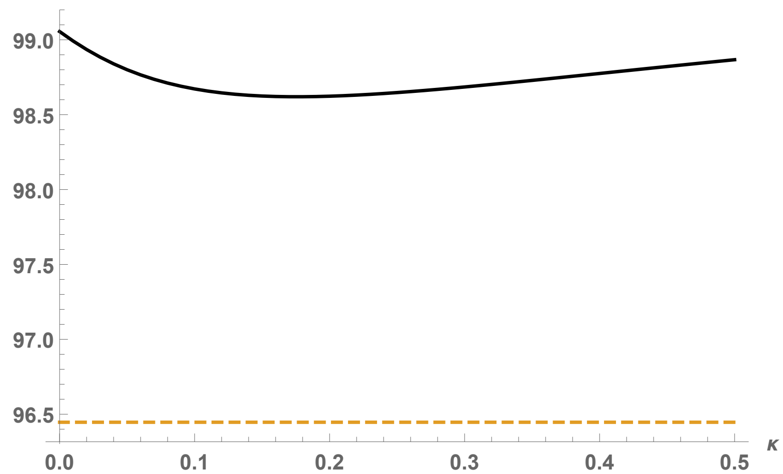


Figure 3: **The effect of κ on firm value in the Intervention equilibrium.** The black line plots the expected period 1 price in the Intervention equilibrium, $p_1 = (1-\theta)p_B + \theta p_S$. The dashed line plots the expected period 1 price in the benchmark case. We assume υ=100, β=25, ω_1=1, ω_2=2, θ=0.1, γ=0.3; $\frac{\varphi}{\alpha}$=0.5, $f[x] = \lambda \exp(-\lambda x)$, and λ=0.1.

however, that in this example firm value is always higher in the intervention equilibrium than in the benchmark case when $\mathcal{A}$ is not present.

The left half of Table 2 summarizes the effects of the various parameters on the existence of the intervention equilibrium and on the equilibrium effectiveness in disciplining the manager, as measured by the ratio $\frac{\delta_{BM}}{\delta_I}$. As noted in column (2), the disciplinary role of intervention is increasing in $\mathcal{A}$'s effectiveness in reducing $\mathcal{M}$'s private benefits of control, $(1-\gamma)$, and is decreasing in $\mathcal{A}$'s effectiveness in restoring the damage to firm value, $(1-\kappa)$. A higher probability of the sell-side liquidity shock, θ, shifts δ_I toward δ_{BM} and therefore decreases the impact of intervention on $\mathcal{M}$'s incentives. That is, higher θ has a negative (positive) impact on $\mathcal{M}$'s incentives when the intervention has a positive (negative) impact on $\mathcal{M}$'s incentives.

Table 2: **Intervention and Exit: Summary.** This table summarizes effects of key parameters on existence and effectiveness of each type of equilibrium. Columns 1 shows the effect of parameters on condition (1). Column 2 shows the effect of parameters on the disciplinary role of the intervention equilibrium, as measured by $\frac{\delta_{BM}}{\delta_I}$. Columns 3 shows the effect of parameters on condition (2). Column 4 shows the effect of parameters on the disciplinary role of the exit equilibrium, as measured by $\frac{\delta_{BM}}{\delta_E}$.

	Intervention		Exit	
	Existence	Effectiveness	Existence	Effectiveness
	(1)	(2)	(3)	(4)
$\mathcal{M}$'s Characteristics				
$1/\lambda$	−	0	+	+/−
β/ω_2	+	0	−	−
ω_1/ω_2	0	0	+	+
$\mathcal{A}$'s Characteristics				
φ/α	+	0	−	0
$(1-\kappa)$	+	−	−	0
$(1-\gamma)$	−	+	0	0
θ	+/−	+/−	+/−	−

Column (1) considers the existence of the intervention equilibrium. The equilibrium is more likely to exist when $\frac{\varphi}{\alpha}$ is higher ($\mathcal{A}$ needs to purchase fewer shares in the open market) and $(1-\kappa)$ is closer to one ($\mathcal{A}$ is effective in restoring the damage). The equilibrium is also more likely to exist as Φ_I increases, that is, when $\mathcal{M}$ is more likely to take the bad action. This happens when β is large (the agency problem is severe), $(1-\gamma)$ is small ($\mathcal{A}$ is less effective in reducing $\mathcal{M}$'s private benefits of control), $(1-\kappa)$ is closer to one ($\mathcal{A}$ is effective in restoring the damage), and when the distribution of δ shifts left. Note that $(1-\kappa)$ positively affects the existence of the intervention equilibrium through two channels (condition (1) and Φ_I).

The impact of the probability of a liquidity shock on existence depends on whether the equilibrium plays a positive or a negative disciplinary role. If $\gamma < \kappa$ and the equilibrium plays a positive disciplinary role, as the probability of a liquidity shock increases, δ_I increases toward δ_{BM} and therefore Φ_I increases as well. As a result, condition (1) is less restrictive. If $\gamma > \kappa$ and the equilibrium plays a negative disciplinary role, as the probability of a liquidity shock increases, δ_I decreases toward δ_{BM} and therefore Φ_I decreases as well. As a result, condition (1) is more restrictive.

Finally, note that the existence and the disciplinary role of this equilibrium does not depend on ω_1, which is one of key parameters that will drive existence and the effectiveness of the exit equilibrium.

2.2. Equilibrium with Exit

Next we construct equilibria in which $\mathcal{A}$ does not intervene when $\mathcal{M}$ takes the bad action (i.e., $\sigma_1^B = 0$). Again, $\Phi_E, \Lambda_E, \delta_E$ represent values of the endogenous variables in the equilibrium.

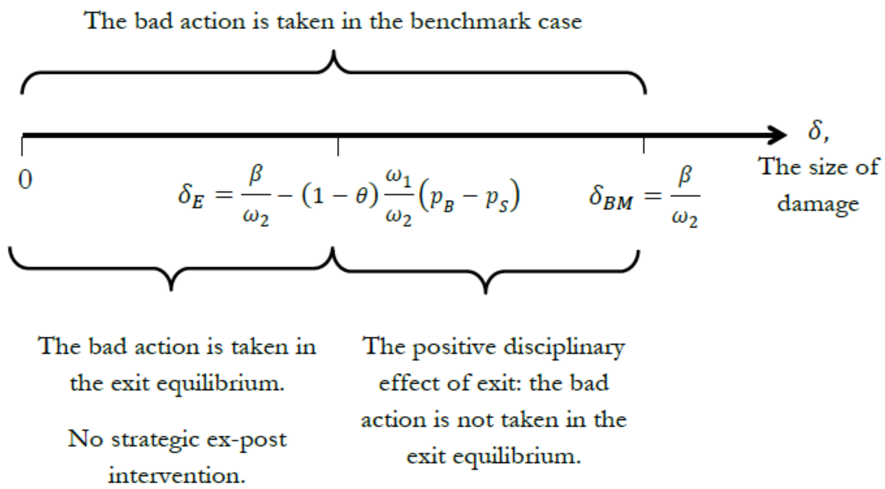


Figure 4: The disciplinary role of Exit.

Proposition 2. *There exists a unique disciplinary equilibrium with* $\sigma_1^B = 0$ *if and only if*

$$\frac{\varphi}{\alpha} < \frac{\kappa}{\Phi_E} \frac{\Phi_E + \bar{\theta}}{1 + \bar{\theta}}, \tag{2}$$

where $\delta_E = \frac{\beta}{\omega_2} - (1-\theta)\frac{\omega_1}{\omega_2}(p_B - p_S)$*,* $\Phi_E = F(\delta_E)$*,* $\Lambda_E = \Phi_E^{-1} E[\mathbb{1}_{\delta<\delta_E}\delta]$ *and equilibrium prices are* $p_B = \upsilon$ *and* $p_S = \upsilon - \Lambda_E \Phi_E \frac{1+\bar{\theta}}{\Phi_E+\bar{\theta}}$*. Equilibrium beliefs are* $\left(\sigma_0^B \geq 0, \sigma_0^H \geq 0, \sigma_0^S = 0; \sigma_1^B = 0, \sigma_1^H = 0, \sigma_1^S = 1\right)$*.*

The impact of exit on $\mathcal{M}$'s incentives is summarized in Figure 4. When $\delta > \delta_{BM}$, $\mathcal{M}$ does not take the bad action even when $\mathcal{A}$ is not present. In this case the damage to firm value is so large that $\mathcal{M}$ prefers to forego the private benefit β. In the intermediate region $\delta_{BM} > \delta > \delta_E$, $\mathcal{A}$'s presence prevents $\mathcal{M}$ from taking the bad action. This is the disciplinary role of exit. Finally, when $\delta < \delta_E$, $\mathcal{M}$ takes the bad action and $\mathcal{A}$ sells his stake.

The right half of Table 2 summarizes the results for the exit equilibrium when the distribution is exponential. As noted in column (4), several parameters affect the disciplinary role of this equilibrium. $\mathcal{M}$ is less likely to take the bad action when ω_1/ω_2 is large ($\mathcal{M}$'s compensation is more dependent on period 1 prices), when β/ω_2 is small (the agency problem is not severe), and when the distribution of δ shifts right. Interestingly, the number of shares owned by $\mathcal{A}$ does not affect the disciplinary role of the exit and intervention equilibria. The chances of a liquidity shock also affect the disciplinary role of the exit equilibrium. Liquidity shocks have a positive impact on the probability that $\mathcal{M}$ takes the bad action because they make prices less informative.

Column (3) summarizes the factors affecting existence of the exit equilibrium. The equilibrium is more likely to exist when $\frac{\varphi}{\alpha}$ is small ($\mathcal{A}$ needs to purchase many shares in the open market in order to intervene) and $(1-\kappa)$ is close to zero ($\mathcal{A}$ is not efficient in restoring the damage). The equilibrium is also more likely to exist as Φ_E decreases, that is, when $\mathcal{M}$ is less likely to take the bad action.

The existence of the exit equilibrium is positively affected by ω_1 because when ω_1 increases, $\mathcal{M}$ is less likely to take the bad action. Thus, when $\mathcal{M}$ is more sensitive to the period 1 prices, the exit equilibrium is more likely to exist. Recall that the existence of the intervention equilibrium does not depend on ω_1.

The effect of θ on the existence of the exit equilibrium could be either positive or negative. On one side, the existence of the exit equilibrium is positively affected by θ because when θ increases, condition (2) is more likely to hold for a given level of the probability of bad action, Φ_E. On the other side, the existence of the exit equilibrium is negatively affected by θ because when θ increases, $\mathcal{M}$ is more likely to take the bad action (i.e., Φ_E increases) and therefore condition (2) is less likely to hold.

2.3. Mixed Strategy Equilibrium

We next show that there can exist an equilibrium in which both intervention and exit have positive probabilities.

Proposition 3. *There is a unique mixed strategy equilibrium if both conditions (1) and (2) are violated. Equilibrium beliefs are* $\left(\sigma_0^B = 1, \sigma_0^H = 0, \sigma_0^S = 0; \sigma_1^B > 0, \sigma_1^H = 0, \sigma_1^S > 1\right)$. *In equilibrium,* $p_B = \upsilon - \kappa\Lambda\Phi\frac{\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)}$ *and* $p_S = \upsilon - \Lambda\Phi\frac{(1-\sigma_1^B)+\bar{\theta}}{\Phi(1-\sigma_1^B)+\bar{\theta}}$*, where* $\delta_{Mixed} = \frac{(\beta/\omega_2)\left[1-(1-\theta)\sigma_1^B(1-\gamma)\right]-(\omega_1/\omega_2)(1-\theta)(1-\sigma_1^B)(p_B-p_S)}{1-(1-\theta)\sigma_1^B(1-\kappa)}$, $\Phi = F(\delta_{Mixed})$, *and* $\Lambda = \Phi^{-1}E[\mathbb{1}_{\delta<\delta_{Mixed}}\delta]$.

This result is reminiscent of several papers which have shown in more complicated contexts that exit and voice can be complementary (e.g., Dasgupta and Piacentino, 2014); in our model it can be thought of as occurring in cases where neither mechanism is strong enough to survive on its own.

2.4. Multiple Equilibria

Multiple equilibria are also possible. One possibility is that both condition (1) and condition (2) are satisfied for their respective values of Φ, in which case we will have one pure strategy equilibrium of each sort. The next proposition provides sufficient conditions for this to occur.

Proposition 4. *Consider a convergent sequence of parameter values with* $\gamma \to 1$. *Suppose for this sequence condition (2) is satisfied, so that there* *is a sequence of exit equilibria. Suppose as well that for the same values of* Φ *condition (1) is satisfied. Then for parameters sufficiently far in the sequence, there are also intervention equilibria.*

In the sequence as constructed, the intervention equilibria are less disciplinary than the exit equilibria; however sequences can also be constructed such that the reverse is true. In section 3.2, we compare the effectiveness of the two equilibria when both occur.[13]

Another possibility is that there is one stable mixed strategy equilibrium and one stable exit equilibrium. This happens when condition (1) is violated (there is no pure intervention equilibrium) and condition (2) holds (there is pure exit equilibrium).[14] If condition (1) is violated, then condition (2) will be violated for sufficiently high values of $\bar{\theta}$.

3. Discussion and Implications

In this section we characterize circumstances when each type of governance is possible and discuss several empirical implications.

3.1. When does only one type of governance work?

We consider circumstances when intervention is the sole equilibrium form of governance.[15] This is the case when condition (1) holds and condition (2) is violated. Three key parameters affect these conditions: $\mathcal{A}$'s

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[13] When there are two pure strategy equilibria, there will also generally be a mixed strategy equilibrium, but it will be unstable.

[14] In this case $G(\sigma_1^B)$, defined in the proof of Proposition 3, crosses X axis for two values of σ_1^B. The solution with the higher value of σ_1^B is a stable equilibrium.

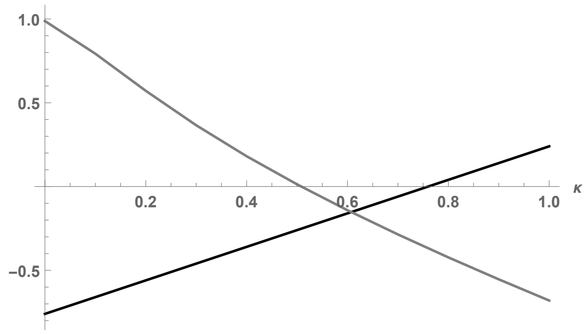
[15] Circumstances when exit is the sole equilibrium form of governance can be analyzed in a similar way.

effectiveness in restoring firm value $(1-\kappa)$, $\mathcal{A}$'s ownership relative to what is needed for intervention $(\frac{\varphi}{\alpha})$, and the probability of the bad action (Φ).

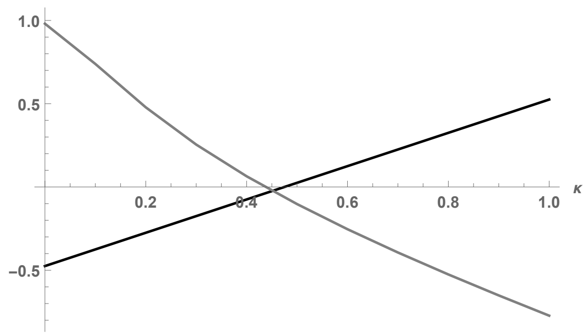
First, consider the role of $\mathcal{A}$'s effectiveness in restoring firm value. As Panel B in Figure 5 shows, Intervention is the only form of governance when κ is small (below 0.45 in that example). In other words, if $\mathcal{A}$ is very effective in restoring firm value ($(1-\kappa)$ is high), then intervention will be the only form of governance. Empirically, this result suggests that when it is common knowledge that there is a shareholder capable of restoring firm value, $\mathcal{M}$'s incentives cannot be affected by $\mathcal{A}$'s threat to liquidate his stake. Brav et al. (2008) document the emergence of activist hedge funds as a class of institutional investors who specializes in intervention and are often effective in increasing the value of firms they target. This empirical regularity is consistent with the prevalence of the intervention type of governance.

As Panels A and B in Figure 5 show, the intervention type of governance is more likely to prevail in equilibrium when $\mathcal{A}$'s ownership is high (as measured by $\frac{\varphi}{\alpha}$). It implies that $\mathcal{M}$'s incentives will be affected by the threat of intervention and not the threat of exit when he learns that an activist shareholder has amassed a substantial toehold. On the other hand, Panels C and B in Figure 5 show that the exit type of governance is more likely to prevail in equilibrium when $\mathcal{A}$'s ownership is small. $\mathcal{M}$ can obtain this information through either Schedule 13F or Schedule 13D.

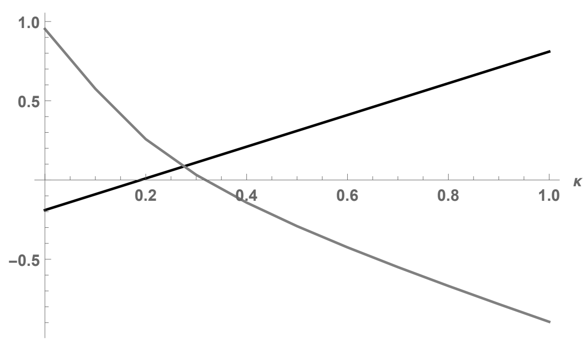
It's worth noting that while $(1-\kappa)$ and $\frac{\varphi}{\alpha}$ increase the chances that the intervention form of governance is the only form of governance in equilibrium, these two parameters have different effects on $\mathcal{M}$'s incentives to take the bad action. Higher $(1-\kappa)$ increases $\mathcal{M}$'s incentives to take the bad action and



(a) $\frac{\varphi}{\alpha}$=0.8



(b) $\frac{\varphi}{\alpha}$=0.5



(c) $\frac{\varphi}{\alpha}$=0.2

Figure 5: When grey (dark) line is above X-axis, the Intervention (Exit) equilibrium exists. We assume υ=100, β=25, ω_1=1, ω_2=2, θ=0.1, γ=0.3, $f[x] = \lambda \exp(-\lambda x)$, and λ=0.1.

$\frac{\varphi}{\alpha}$ does not affect $\mathcal{M}$'s incentives to take the bad action.

Finally, the intervention type of governance is more likely to be the equilibrium form of governance when $\mathcal{M}$ is more likely to take the bad action in the intervention equilibrium and when $\mathcal{M}$ is more likely to take the bad action in the exit equilibrium.

3.2. When both Exit and Intervention discipline the manager

We next characterize circumstances when both exit and intervention types of governance can discipline $\mathcal{M}$. Both types of governance happen when condition (1) and condition (2) hold. In those circumstances where both types governance can happen, what considerations increase effectiveness of intervention vs. exit?

Effectiveness of each governance mechanism is measured by expected period 1 price in each equilibrium:

$$p_I = (1-\theta)p_{BI} + \theta p_{SI} \tag{3}$$

and

$$p_E = (1-\theta)(\Phi_E p_{SE} + (1-\Phi_E)p_{BE}) + \theta p_{SE}, \tag{4}$$

where $p_{BI} = \upsilon - \kappa\Lambda_I\Phi_I$, $p_{SI} = \upsilon - \Lambda_I\Phi_I$ (see Proposition 1 for further details), $p_{BE} = \upsilon$, and $p_{SE} = \upsilon - \Lambda_E\Phi_E\frac{1+\bar{\theta}}{\Phi_E+\theta}$ (see Proposition 2 for further details).

Thus, in order to compare the effectiveness of two governance mechanisms one needs to analyze equilibrium prices and stock liquidity. In the next section we perform such analysis.

3.3. Equilibrium prices, stock liquidity, and corporate governance

In this section we compare endogenously determined bid and ask prices as well as expected period 1 prices in each model, analyze forces that affect bid-ask spread, and study the relation between measured stock liquidity and two corporate governance mechanisms.

In the intervention equilibrium, $p_B = \upsilon - \kappa\Lambda_I\Phi_I$ and $p_S = \upsilon - \Lambda_I\Phi_I$. Both prices decline when $\mathcal{M}$ is more likely to take the bad action (higher Φ_I) and when the expected damage to firm value is large (higher Λ_I). Figure 6 plots equilibrium prices as function of θ when intervention plays a positive disciplinary role ($\gamma < \kappa$). We see that prices decrease when $\mathcal{A}$ is more likely to experience a sell-side liquidity shock. Thus, firm value is decreasing in θ when $\gamma < \kappa$.

In the intervention equilibrium, the bid-ask spread is $p_B - p_S = (1 - \kappa)\Lambda_I\Phi_I$. It is positive as long as $\mathcal{A}$ is effective in restoring the damage, $(1-\kappa) > 0$. Thus, $\mathcal{A}$'s activism skills are the source of information asymmetry in this equilibrium (see also Collin-Dufresne and Fos, 2014). Interestingly, the bid-ask spread is wider when $\mathcal{A}$ is more likely to experience a sell-side liquidity shock. This is because in the disciplinary Intervention equilibrium higher θ increases chances that $\mathcal{M}$ takes the bad action. Thus, Figure 6 reveals that lower likelihood of sell-side liquidity shocks leads to higher measured stock liquidity (narrower bid-ask spread) and higher firm value (higher p_1), implying a positive correlation between these two endogenously determined values.

Corollary 1. *In a disciplinary intervention equilibrium ($\gamma < \kappa$), lower θ leads to higher measured stock liquidity (narrower bid-ask spread) and higher*

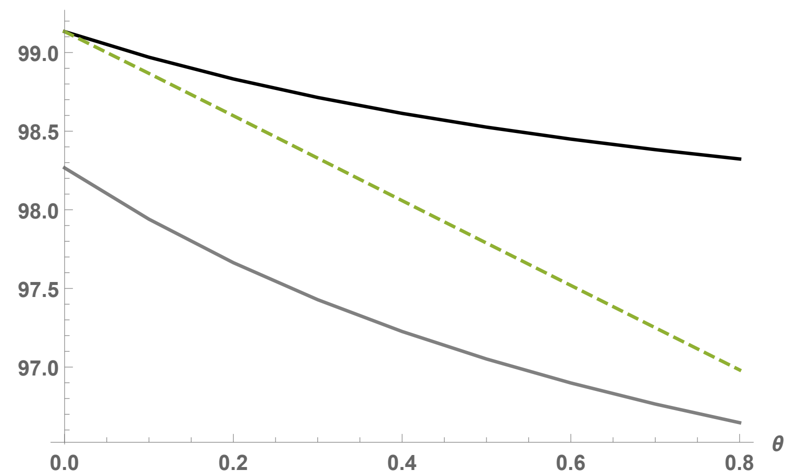


Figure 6: **The effect of θ on prices in the Intervention equilibrium.** The black line plots period 1 buy price, p_B. The grey line plots period 1 sell price, p_S. The dashed line plots the expected period 1 price, $p_1 = (1-\theta)p_B + \theta p_S$. We assume υ=100, β=25, ω_1=1, ω_2=2, γ=0.3; $\frac{\varphi}{\alpha}$=0.8, κ=0.5, $f[x] = \lambda \exp(-\lambda x)$, and λ=0.1.

firm value.

The model shows that if the intervention equilibrium is disciplinary (panel A in Figure 4) then, conditional on having a block, $\mathcal{A}$ is *less* likely to intervene when θ decreases and as a result of smaller θ the measured liquidity increases. This happens because when θ decreases, the intervention mechanism becomes more disciplinary (δ_I is lower) and therefore fewer interventions take place in equilibrium. This prediction finds support in Edmans et al. (2013), who show that conditional on having a large block (i.e., high $\frac{\varphi}{\alpha}$, which implies the existence of the Intervention equilibrium), activists are more likely to file (passive) Schedule 13G vs. (active) Schedule 13D when measured liquidity increases. Similarly, Back et al. (2013) find that higher stock liquidity leads to fewer interventions by large blockholders. Interestingly, while the authors interpret the evidence as an indication that liquidity is harmful for the "voice" form of governance, our model suggests that that evidence is also consistent with an equilibrium in which the *threat* of intervention plays a stronger disciplinary role:

Corollary 2. *In a disciplinary intervention equilibrium ($\gamma < \kappa$) in which $\mathcal{A}$ holds a substantial toehold ($\frac{\varphi}{\alpha}$ is close to one), lower θ leads to narrower bid-ask spreads and smaller probability of intervention.*

In the exit equilibrium, the formulas in Proposition 2 show that when $\theta = 0$, $p_S = \upsilon - \Lambda_E$, reflecting that in equilibrium $\mathcal{A}$ sells when the bad action is taken. When $\theta > 0$, $p_S > \upsilon - \Lambda_E$ because liquidity shocks force $\mathcal{A}$ to sell not only when the bad action is taken, but also when the bad action is not taken. Figure 7 shows the effects of θ on equilibrium prices in the exit equilibrium.

When the chances of forced liquidity sale increase, the price at which $\mathcal{A}$ can sell increases. The difference between the prices (i.e., bid-ask spread) narrows, corresponding to higher measured stock liquidity. Interestingly, when θ is high the equilibrium has lower bid-ask spread as well as lower firm value, implying a negative correlation between endogenously determined firm value and bid-ask spread. When related to the recent empirical literature on the positive relation between measured stock liquidity and firm value (e.g., Edmans et al., 2013; Bharath et al., 2013; Fos, 2015), this finding indicates that forces other than discipline through exit may drive the relation between firm value and stock liquidity.

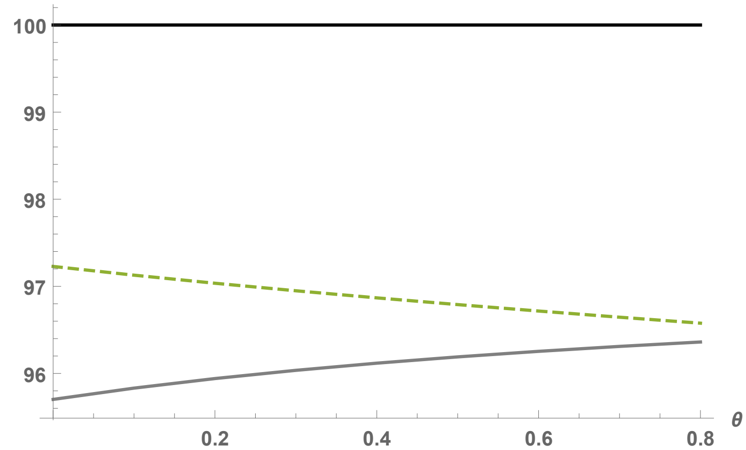


Figure 7: **The effect of θ on prices in the Exit equilibrium.** The black line plots period 1 buy price, p_B. The grey line plots period 1 sell price, p_S. The dashed line plots the expected period 1 price, $p_1 = (1-\theta)(\Phi_E p_S + (1-\Phi_E)p_B) + \theta p_S$. We assume υ=100, β=25, ω_1=1, ω_2=2, γ=0.3, $\frac{\varphi}{\alpha}$=0.2, κ=0.5, $f[x] = \lambda \exp(-\lambda x)$, and λ=0.1.

Corollary 3. *In the exit equilibrium, higher θ leads to higher measured stock liquidity (narrower bid-ask spread) and lower firm value.*

4. Initial toehold

In this section we discuss the formation of the initial toehold, φ. The initial toehold plays an important role in the model because it determines what type of corporate governance will prevail in equilibrium. To examine this issue, we provide a simple extension to the model which endogenizes the activist's choice of size of toehold. In this section we will use the term "effective" to refer to the governance structure that yields the higher expected value of the firm in equilibrium.[16]

Suppose at the beginning of period 0 $\mathcal{A}$ chooses how many shares φ to buy. The market does not observe $\mathcal{A}$'s purchase or its size; instead there is an expectation that with probability $1-\nu$ a purchase will be made by $\mathcal{A}$ and with probability ν a purchase will be made by a non-monitoring investor. If $\mathcal{A}$ does not purchase, the value of the firm is p_{BM}. In effect, higher values ν correspond to a more liquid period 0 market; $\mathcal{A}$'s purchase can be hidden more effectively in the sea of non-monitor purchases. The actual price on the market in period 0, denoted p_0, will reflect the activism role played by $\mathcal{A}$ as well as the market's expectation that he is participating. In other words, $p_0(\nu) = (1-\nu)p_1(\varphi) + \nu p_{BM}$, where $p_1(\varphi)$ is the expected price in period 1.

After the initial trade occurs, $\mathcal{A}$'s ownership stake (φ) becomes common knowledge. This assumption is motivated by the fact that market participants can use Schedule 13F, Schedule 13D, and Schedule 13G filings to infer changes in stock ownership. It is also assumed that holding φ shares involves private cost $C(\varphi) = \phi\frac{\varphi^2}{2}$ for $\mathcal{A}$. For example, this cost could correspond to lower diversification of $\mathcal{A}$'s portfolio.[17]

Recall that conditions (1) and (2) determine the existence of intervention and exit equilibrium, respectively. If φ is close to α, condition (1) is more likely to hold and condition (2) is less likely to hold. Similarly, if φ is close to zero, condition (1) is less likely to hold and condition (2) is more

[16]While "effectiveness" is a natural criterion, it does not necessarily equate to Pareto optimality, since we are ignoring not only the costs of the activist to acquiring the initial toehold (discussed below) but also the payoff to the manager.

[17]The analysis can be extended to consider the possibility that higher φ increases the likelihood that $\mathcal{A}$ faces a liquidity shock in period 1.

likely to hold. Given α, let φ_I^* and φ_E^* represent the boundary values for the two conditions, so that values φ above $\max(\varphi_I^*, \varphi_E^*)$ lead to the intervention equilibrium, and values below $\min(\varphi_I^*, \varphi_E^*)$ lead to the exit equilibrium. In equilibrium, then, φ affects firm value by determining what type of governance will prevail. We will focus on the case where $\varphi_I^* < \varphi_E^*$, so that for intermediate values of φ both types of equilibria exist.[18]

What size of the initial toehold will $\mathcal{A}$ choose? For simplicity assume that when φ is consistent with multiple equilibria, the equilibrium selected is the one most preferred by $\mathcal{A}$.[19] $\mathcal{A}$'s choice of φ is affected by several factors. $\mathcal{A}$'s expected profits are (after dropping factors not affected by ϕ):

$$V(\varphi; p_1) = \varphi\nu(p_1(\varphi) - p_{BM}) - \phi\frac{\varphi^2}{2}. \tag{5}$$

Note that as long as $\nu = 0$, $\mathcal{A}$ prefers $\varphi = 0$. In other words, if $\mathcal{A}$ needs to purchase φ shares in the open market at a price that reflects $\mathcal{A}$'s impact of firm value, privately-optimal initial stake size will be zero in the absence of liquidity trading. When $\nu > 0$, $\mathcal{A}$ profits from liquidity trading because prices do not fully reflect $\mathcal{A}$'s impact of firm value.

Before we derive the optimal size of the initial toehold for $\mathcal{A}$, we characterize $\mathcal{A}$'s choice of φ for a given level of period 1 prices.

Lemma 2. *$\mathcal{A}$'s choice of φ for a given level of period 1 prices is* $\varphi_{\mathcal{A}} =$

[18] The opposite possibility leads to mixed strategy equilibria when φ is in the intermediate range; this possibility can be analyzed in a similar fashion.

[19] An interesting possible technical extension would be to use the structure of global games to develop a selection criterion. In this case, the boundary generated by the selection criterion substitutes for φ^*'s in the following analysis.

$\frac{1}{\phi}\nu(p_1 - p_{BM})$. *$\mathcal{A}$'s expected profit is* $V(\varphi_{\mathcal{A}}; p_1) = \frac{1}{2\phi}\nu^2(p_1 - p_{BM})^2$.

In other words, $\mathcal{A}$ will choose higher φ when liquidity trading is large (ν), the cost of holding the block is small (ϕ), and $\mathcal{A}$'s impact on firm value is large ($p_1 - p_{BM}$).[20] The following propositions characterize $\mathcal{A}$'s optimal φ, while taking into account the effect of φ on the form of governance that will prevail in equilibrium. First consider the case when intervention is the effective type of governance:

Proposition 5. *Suppose* $p_1^I > p_1^E$. *Let* $\overline{\varphi_I}$ *be such that* $V(\overline{\varphi_I}; p_1^I) = V(\varphi_{\mathcal{A}}; p_1^E)$. *$\mathcal{A}$ will choose* $\varphi_{\mathcal{A}} \geq \varphi_I^*$ *as long as* $\varphi_I^* \leq \overline{\varphi_I}$. *In this case intervention will be the equilibrium type of governance. If* $\varphi_I^* > \overline{\varphi_I}$, *$\mathcal{A}$ will choose* $\varphi_{\mathcal{A}} = \varphi_{\mathcal{A}}^E$. *In this case exit will be the equilibrium type of governance.*

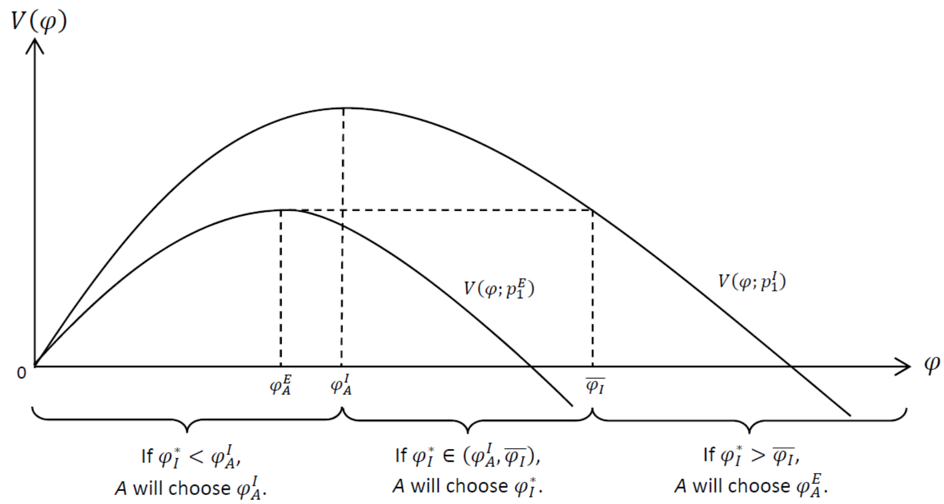
The intuition behind Proposition 5 is presented in Panel A of Figure 8.

When $\varphi_I^* < \varphi_{\mathcal{A}}^I$, $\mathcal{A}$ finds it optimal to choose $\varphi_{\mathcal{A}}^I$ that maximizes $\mathcal{A}$'s profits in the intervention equilibrium. Thus, intervention is the equilibrium type of governance and because $\varphi_I^* < \varphi_{\mathcal{A}}^I$, $\mathcal{A}$'s private choice leads to the optimal type of governance.

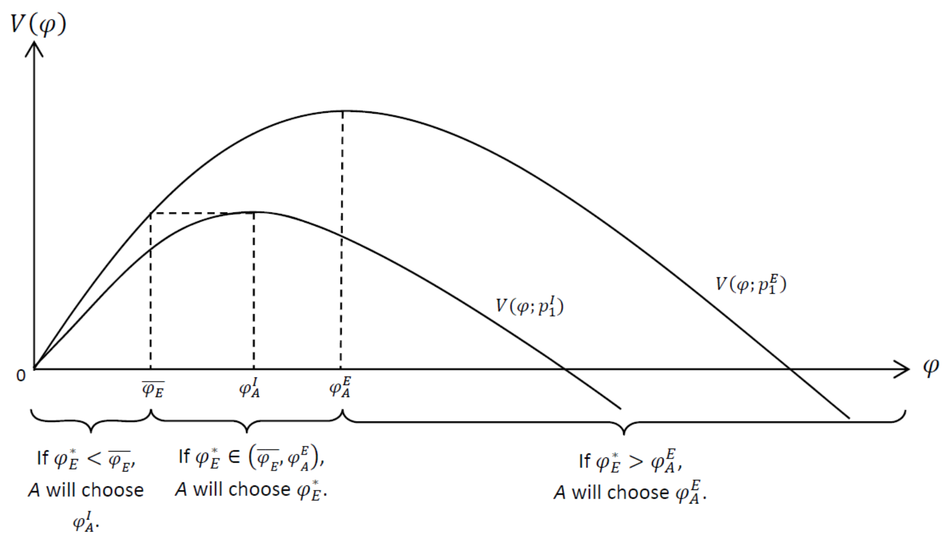
When $\varphi_I^* \in (\varphi_{\mathcal{A}}^I, \overline{\varphi_I})$, $\varphi_{\mathcal{A}}^I$ is not large enough to maintain the intervention equilibrium. $\mathcal{A}$ realizes that he needs to choose a stake large enough to maintain prices from the intervention equilibrium, and so chooses the lowest possible φ such that intervention is the equilibrium type of governance, namely φ_I^*.

In the above two cases $\mathcal{A}$'s private choice leads to adoption of the effective type of governance. In contrast, when $\varphi_I^* > \overline{\varphi_I}$, $\mathcal{A}$ does not find it

[20] A sufficient condition to guarantee that $\varphi_{\mathcal{A}}$ does not exceed α is $\phi > \frac{\upsilon - p_{BM}}{\alpha}$.



(a) $p_1^I > p_1^E$: Intervention is socially optimal type of governance



(b) $p_1^E > p_1^I$: Exit is socially optimal type of governance

Figure 8: $\mathcal{A}$'s choice of φ.

optimal to build a toehold which is large enough to support the intervention equilibrium. As a result, $\mathcal{A}$ will prefer to switch to the exit equilibrium and will select the reduced level of toehold $\varphi_{\mathcal{A}} = \varphi_{\mathcal{A}}^E$. In this case $\mathcal{A}$'s private choice leads to adoption of the less effective type of governance.

Next consider the case when exit is the effective type of governance:

Proposition 6. *Suppose $p_1^E > p_1^I$. Let $\overline{\varphi_E}$ be such that $V(\overline{\varphi_E}; p_1^E) = V(\varphi_{\mathcal{A}}; p_1^I)$. $\mathcal{A}$ will choose $\varphi_{\mathcal{A}} \leq \varphi_E^*$ as long as $\varphi_E^* \geq \overline{\varphi_E}$. In this case exit will be the equilibrium type of governance. If $\varphi_E^* < \overline{\varphi_E}$, $\mathcal{A}$ will choose $\varphi_{\mathcal{A}} = \varphi_{\mathcal{A}}^I$. In this case intervention will be the equilibrium type of governance.*

The intuition behind proposition 6 is presented in panel B. When $\varphi_E^* > \varphi_{\mathcal{A}}^E$, $\mathcal{A}$ chooses a block size small enough to maintain the exit equilibrium. In this case $\mathcal{A}$'s private choice leads to the optimal type of governance. When $\varphi_E^* \in (\overline{\varphi_E}, \varphi_{\mathcal{A}}^E)$, $\mathcal{A}$ realizes that $\varphi_{\mathcal{A}}^E$ is too large to maintain the exit equilibrium. Therefore, $\mathcal{A}$ will choose the highest possible φ such that exit is the equilibrium type of governance, namely φ_E^*.

In the above two cases $\mathcal{A}$'s private choice leads to adoption of the effective type of governance. In contrast, when $\varphi_E^* < \overline{\varphi_E}$, $\mathcal{A}$'s private choice leads to adoption of the less effective governance. Instead $\mathcal{A}$ finds it optimal to build a toehold too large to support the exit equilibrium, choosing instead the profit maximizing intervention equilibrium $\varphi_{\mathcal{A}} = \varphi_{\mathcal{A}}^I$.

In general, the intervention equilibrium requires a large initial toehold, and the exit equilibrium requires a small initial toehold. Thus depending on which of the two types of governance is more effective, factors that encourage larger initial toeholds (liquid markets in the initial period, lower costs to amassing the initial toehold) can increase *or* reduce the effectiveness of governance. For example, *smaller* liquidity trading and larger cost of holding the toehold are beneficial when the effective equilibrium is the exit equilibrium.

5. Conclusion

In this paper we have developed a model to compare two categories of disciplinary mechanisms used by activist shareholders: intervention and exit. We have derived predictions as to when one or the other is more likely to be available and more likely to be effective in disciplining the manager. In addition to the considerations which have been frequently alluded to in studies of the individual categories (size of share needed for a toehold and other costs of engaging in activism, relevant for intervention; sensitivity of managerial compensation to short run share price, relevant for exit), we have also examined factors which are of importance in both intervention and exit mechanisms.

When a manager engages in destructive behavior, intervention can have two effects: it can restore firm value, or it can reduce managerial benefit from the behavior. As we have seen, because of the interactions with the behavior of the manager, these two effects lead to different implications for effectiveness of intervention, and the likelihood that it is observed instead of exit activity. Empirically, these predictions could be tested using data on institutional ownership. Activist shareholders are likely to have skills in recovering firm value (e.g., Brav et al., 2008). Moreover, activist shareholders are able to impose private costs on management when proxy contests are concerned (e.g., Fos and Tsoutsoura, 2014). Changes in beneficial ownership of activist shareholders could therefore be used to study what type of governance is effective.[21]

Finally, liquidity has a complicated effect on the forms of governance. Our model suggests that empiricists need to be more cautious when studying implications of stock liquidity on corporate governance—first, because the empirical measures of stock liquidity, such as bid-ask spreads are endogenous to the equilibrium trading decisions made by activist shareholders. But even if the underlying liquidity shocks faced by activists are themselves exogenous, we show depending on the type of Intervention equilibrium (disciplinary or not), the effects of liquidity shocks can be different.

[21]For example, one could argue that an increase in activist hedge funds ownership enhances the role of the intervention mechanism. However, endogeneity of activist ownership suggests that an extraordinary caution is required when such empirical analysis is performed.

References

Admati, A., Pfleiderer, P., May 2009. The "Wall Street Walk" and shareholder activism: Exit as a form of voice. The Review of Financial Studies 22 (7), 2645–2685.

Admati, A. R., Pfleiderer, P., Zechner, J., 1994. Large shareholder activism, risk sharing, and financial market equilibrium. Journal of Political Economy 102 (6), 1097–1130.

Back, K., Li, T., Ljungqvist, A., 2013. Liquidity and governance, National Bureau of Economic Research working paper, No. w19669.

Baumol, W., Panzar, J., Willig, R., 1988. Contestable Markets and the Theory of Industry Structure, revised edition Edition. Harcourt Brace Jovanovish, Publishers.

Bharath, S. T., Jayaraman, S., Nagar, V., 2013. Exit as governance: An empirical analysis. The Journal of Finance 68 (6), 2515–2547.

Bolton, P., von Thadden, E.-L., February 1998. Blocks, liquidity, and corporate control. The Journal of Finance 53 (1), 1–25.

Brav, A., Jiang, W., Partnoy, F., Thomas, R., August 2008. Hedge fund activism, corporate governance, and firm performance. The Journal of Finance 63 (4), 1729–1775.

Burkart, M., Gromb, D., Panunzi, F., 1997. Large shareholders, monitoring, and the value of the firm. The Quarterly Journal of Economics 112 (3), 693–728.

Collin-Dufresne, P., Fos, V., 2014. Shareholder activism, informed trading, and stock prices, working paper.

Collin-Dufresne, P., Fos, V., 2015. Do prices reveal the presence of informed trading? Journal of Finance, Forthcoming.

Dasgupta, A., Piacentino, G., 2014. The wall street walk when blockholders compete for flows. The Journal of Finance, forthcoming.

Edmans, A., December 2009. Blockholder trading, market efficiency, and managerial myopia. The Journal of Finance 64 (6), 2481–2513.

Edmans, A., 2013. Blockholders and corporate governance, working paper.

Edmans, A., Fang, V., Zur, E., 2013. The effect of liquidity on governance. Review of Financial Studies 26 (6), 1443–1482.

Edmans, A., Manso, G., 2011. Governance through trading and intervention: A theory of multiple blockholder. Review of Financial Studies 24 (7), 2395–2428.

Faure-Grimaud, A., Gromb, D., 2004. Public trading and private incentives. Review of financial Studies 17 (4), 985–1014.

Fluck, Z., 1999. The dynamics of the management-shareholder conflict. Review of Financial Studies 12 (2), 379–404.

Fos, V., 2015. The disciplinary effects of proxy contests. Management Science, forthcoming.

Fos, V., Jiang, W., 2015. Out-of-the-money CEOs: Inferring private control premium from CEO option exercises. Review of Financial Studies, forthcoming.

Fos, V., Tsoutsoura, M., 2014. Shareholder democracy in play: Career consequences of proxy contests. Journal of Financial Economics 114 (2), 316–340.

Grossman, S. J., Hart, O. D., 1980. Takeover bids, the free-rider problem, and the theory of the corporation. The Bell Journal of Economics 11 (1), 42–64.

Kahn, C., Winton, A., February 1998. Ownership structure, speculation, and shareholder intervention. The Journal of Finance 53 (1), 99–129.

Kyle, A., Vila, J.-L., 1991. Noise trading and takeovers. RAND Journal of Economics 22 (1), 54–71.

Levit, D., 2012. Soft shareholder activism, working paper, Wharton.

Maug, E., February 1998. Large shareholders as monitors: Is there a trade-off between liquidity and control? The Journal of Finance 53 (1), 65–98.

Noe, T. H., 2002. Investor activism and financial market structure. Review of Financial Studies 15 (1), 289–318.

Scharfstein, D., 1988. The disciplinary role of takeovers. The Review of Economic Studies 55 (2), 185–199.

Shleifer, A., Vishny, R. W., 1986. Large shareholders and corporate control. Journal of Political Economy 94 (3), 461–488.

Stein, J. C., 1988. Takeover threats and managerial myopia. Journal of Political Economy 96 (1), 61–80.

Zwiebel, J., 1996. Dynamic capital structure under managerial entrenchment. The American Economic Review 86 (5), 1197–1215.

Proofs

Proof of Lemma 1.

$\mathcal{A}$'s profits, as measured by $\pi - \varphi\upsilon$, are as follows:

	$a = 0$ (no damage)	$a = 1$ (damage)
Buy	$(\alpha-\varphi)\kappa\Lambda\{\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)\sigma_0^B}, 0\}$	$-\alpha\kappa\Lambda + (\alpha-\varphi)\kappa\Lambda\{\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)\sigma_0^B}, 0\}$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda\frac{\Phi\sigma_1^S+\bar{\theta}\Phi}{\Phi\sigma_1^S+(1-\Phi)\sigma_0^S+\bar{\theta}}$	$-\varphi\Lambda\frac{\Phi\sigma_1^S+\bar{\theta}\Phi}{\Phi\sigma_1^S+(1-\Phi)\sigma_0^S+\bar{\theta}}$

where in the case of each bracketed expression, the second term is to be used in case the denominator of the first term is zero.

Since $\Phi > 0$, $\pi_0^B - \varphi\upsilon > \pi_0^S - \varphi\upsilon$ implies $\sigma_0^S = 0$. Similarly, $\pi_1^S - \varphi\upsilon > \pi_1^H - \varphi\upsilon$ implies $\sigma_1^H = 0$ if $\Phi < 1$. A sufficient condition for $\Phi < 1$ is that the support of the distribution include sufficiently high values of δ such that the manager is uninterested in taking the action. For example, it is sufficient to assume $F(\beta/\omega_2) < 1$.

Proof of Proposition 1.

When $\sigma_1^B > 0$, $\mathcal{A}$'s profits can be simplified as follows:

	$a = 0$ (no damage)	$a = 1$ (damage)
Buy	$(\alpha-\varphi)\kappa\Lambda\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)\sigma_0^B}$	$-\alpha\kappa\Lambda + (\alpha-\varphi)\kappa\Lambda\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)\sigma_0^B}$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda\frac{\Phi\sigma_1^S+\bar{\theta}\Phi}{\Phi\sigma_1^S+\bar{\theta}}$	$-\varphi\Lambda\frac{\Phi\sigma_1^S+\bar{\theta}\Phi}{\Phi\sigma_1^S+\bar{\theta}}$

In this case $\sigma_0^B = 1$, $\sigma_0^H = 0$, $\sigma_0^S = 0$, and $\sigma_1^H = 0$. $\mathcal{A}$'s profits can be further simplified as follows:

	$a=0$ (no damage)	$a=1$ (damage)
Buy	$(\alpha-\varphi)\kappa\Lambda\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)}$	$-\alpha\kappa\Lambda+(\alpha-\varphi)\kappa\Lambda\frac{\Phi\sigma_1^B}{\Phi\sigma_1^B+(1-\Phi)}$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda\frac{\Phi\sigma_1^S+\bar{\theta}\Phi}{\Phi\sigma_1^S+\bar{\theta}}$	$-\varphi\Lambda\frac{\Phi\sigma_1^S+\bar{\theta}\Phi}{\Phi\sigma_1^S+\bar{\theta}}$

Condition 1 follows from comparing π_1^B and π_1^S when $\sigma_1^B=1$.

Given the beliefs, $\mathcal{M}$ expects $P_1(a=0)=P_1(a=1)=(1-\theta)p_B+\theta p_S$. Moreover, in equilibrium $\mathcal{M}$ consumes private benefits $\beta(\theta+(1-\theta)\gamma)$ if $a=1$. Thus, if $\mathcal{M}$ does not take the action, his expected utility is $\omega_1\left((1-\theta)p_B+\theta p_S\right)+\omega_2 \upsilon$. If $\mathcal{M}$ takes the action, his expected utility is $\omega_1\left((1-\theta)p_B+\theta p_S\right)+\omega_2(\upsilon-\widetilde{\delta}(\theta+(1-\theta)\kappa))+\beta(\theta+(1-\theta)\gamma)$. The cutoff point is therefore $\delta_I=\beta/\omega_2\frac{\theta+(1-\theta)\gamma}{\theta+(1-\theta)\kappa}$.

Proof of Proposition 2.

If $\sigma_1^B=0$, $\sigma_1^S=1$ and $\mathcal{A}$'s profits can be simplified as follows:

	$a=0$ (no damage)	$a=1$ (damage)
Buy	0	$-\alpha\kappa\Lambda$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda\frac{\Phi+\bar{\theta}\Phi}{\Phi+\bar{\theta}}$	$-\varphi\Lambda\frac{\Phi+\bar{\theta}\Phi}{\Phi+\bar{\theta}}$

Condition 2 follows from comparing π_1^B and π_1^S.

Given the beliefs, $\mathcal{M}$ expects $P_1(a=0)=(1-\theta)p_B+\theta p_S$ and $P_1(a=1)=p_S$. Note that we assumed that stock price is p_B is $\mathcal{M}$ does not take the action and $\mathcal{A}$ holds φ shares. Moreover, in equilibrium $\mathcal{M}$ consumes private benefits β if $a=1$. Thus, if $\mathcal{M}$ does not take the action, his expected utility is $\omega_1\left(p_B-\theta(p_B-p_S)\right)+\omega_2\upsilon$. If $\mathcal{M}$ takes the action, his expected utility is

$\omega_1 p_S + \omega_2(\upsilon - \widetilde{\delta}) + \beta$. The cutoff point is therefore $\delta_E = \frac{\beta}{\omega_2} - (1-\theta)\frac{\omega_1}{\omega_2}(p_B - p_S)$, where $p_B - p_S = \Lambda\Phi\frac{1+\bar{\theta}}{\Phi+\bar{\theta}}$.

Proof of Proposition 3.

We want to construct an equilibrium with $\sigma_1^B > 0$ and $\sigma_1^S > 0$. When $\sigma_1^B > 0$, $\sigma_0^B = 1$. Consider $G(\sigma_1^B) \equiv \pi_1^B - \pi_1^S$ when $\sigma_1^B \in (0,1)$:

$$G(\sigma_1^B) = -\alpha\kappa\Lambda + (\alpha - \varphi)\kappa\Phi\Lambda\frac{\sigma_1^B}{\Phi\sigma_1^B + (1-\Phi)} + \varphi\Phi\Lambda\frac{(1-\sigma_1^B)+\bar{\theta}}{\Phi(1-\sigma_1^B)+\bar{\theta}}.$$

For there to be an equilibrium with $\sigma_1^B \in (0,1)$ for a given Φ, it is necessary and sufficient that $G(\sigma_1^B) = 0$ at some $\sigma_1^B > 0$. Eliminating denominators yields a quadratic function:

$$\begin{aligned}
&-\alpha\kappa(\Phi\sigma_1^B + (1-\Phi))(\Phi(1-\sigma_1^B)+\bar{\theta}) \\
&+(\alpha-\varphi)\kappa\Phi\sigma_1^B(\Phi(1-\sigma_1^B)+\bar{\theta}) \\
&+\varphi\Phi((1-\sigma_1^B)+\bar{\theta})(\Phi\sigma_1^B + (1-\Phi)),
\end{aligned}$$

for which the quadratic coefficient is $-(1-\kappa)\varphi\Phi^2 < 0$. The function is continuous and by conjecture, $\lim\limits_{\sigma_1^B \to 0} > 0$ and $\lim\limits_{\sigma_1^B \to 1} < 0$. Thus there are an odd number of crossings of the horizontal axis. Since the function is quadratic there is exactly one crossing $\in (0,1)$. The values of p_B and p_S are immediate from their definitions. Given the beliefs, $\mathcal{M}$ expects $P_1(a=0) = (1-\theta)p_B + \theta p_S$ and $P_1(a=1) = (1-\theta)\left[\sigma_1^B p_B + (1-\sigma_1^B)p_S\right] + \theta p_S$. Moreover, in equilibrium $\mathcal{M}$ is expected to consume private benefits $\beta\left[1-(1-\theta)\sigma_1^B(1-\gamma)\right]$. Thus, if $\mathcal{M}$ does not take the action, his expected utility is $\omega_1 P_1(a=0) + \omega_2\upsilon$. If $\mathcal{M}$ takes the action, his expected utility is

$\omega_1 P_1(a=1) + \omega_2(\upsilon - \widetilde{\delta}\left[1-(1-\theta)\sigma_1^B(1-\kappa)\right]) + \beta\left[1-(1-\theta)\sigma_1^B(1-\gamma)\right]$.
The cutoff point is therefore

$$\delta_{Mixed} = \frac{(\beta/\omega_2)\left[1-(1-\theta)\sigma_1^B(1-\gamma)\right] - (\omega_1/\omega_2)(1-\theta)(1-\sigma_1^B)(p_B - p_S)}{1-(1-\theta)\sigma_1^B(1-\kappa)}.$$

Proof of Proposition 4

Suppose we have a sequence of parameter values satisfying the conditions of the proposition. Then those sequences yield a sequence of exit equilibria. However they do not necessarily yield a sequence of intervention equilibria, since the value Φ that applies for the exit equilibrium will not in general be the same value for the intervention equilibrium. Nonetheless, examination of the condition (1) demonstrates that if it is satisfied for one value of Φ, it is also satisfied for all higher values of Φ. Thus if the conditions defining the cutoff in the case of intervention are less stringent than in the case of the corresponding exit equilibrium, we in fact have an intervention equilibrium as well. By the earlier propositions, the intervention cutoff is

$$\delta_I = \beta/\omega_2 \frac{\theta + (1-\theta)\gamma}{\theta + (1-\theta)\kappa}$$

and the exit cutoff is

$$\delta_E = \frac{\beta}{\omega_2} - (1-\theta)\frac{\omega_1}{\omega_2}(p_B - p_S),$$

where

$$p_B - p_S = \Lambda\Phi\frac{1+\bar{\theta}}{\Phi + \bar{\theta}}.$$

Thus $\delta_I > \delta_E$ is equivalent to

$$\beta/\omega_2 \frac{\theta + (1-\theta)\gamma}{\theta + (1-\theta)\kappa} > \frac{\beta}{\omega_2} - (1-\theta)\frac{\omega_1}{\omega_2}(p_B - p_S).$$

In the limit as $\gamma \to 1$ the expression is:

$$(1-\theta)(p_B - p_S) > \frac{\beta}{\omega_1}\left[1 - \frac{1}{\theta + (1-\theta)\kappa}\right].$$

The inequality holds in the limit because $p_B > p_S$ and $\theta + (1-\theta)\kappa < 1$.