The Labor Market for Directors and Externalities in Corporate Governance

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October 16, 2014

Abstract

This paper studies how directors' reputational concerns affect board structure, corporate governance, and firm value. In our setting, directors affect their firms' governance, and governance, in turn, affects firms' demand for new directors. Whether the labor market rewards a shareholder-friendly or management-friendly reputation is determined in equilibrium and depends on aggregate governance. We show that directors' desire to be invited to other boards creates strategic complementarity of corporate governance across firms. Directors' reputational concerns amplify the governance system: strong systems become stronger and weak systems become weaker. We derive implications for multiple directorships, board size, transparency, and board independence.

Keywords: board of directors, corporate governance, reputation, externalities, strategic complementarity, transparency

JEL Classification Numbers: D62, D71, D82, D83, G34, G38, J20

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*Doron Levit is from the University of Pennsylvania, Wharton School, Finance Department. Nadya Malenko is from the Carroll School of Management at Boston College, Finance Department. We thank Jeffrey Coles, David Dicks (discussant), Vincent Glode, Itay Goldstein, Todd Gormley, Barney Hartman-Glaser (discussant), Oguzhan Karakas, Andrey Malenko, Egor Matveyev (discussant), Christian Opp, Giorgia Piacentino (discussant), Jun "QJ" Qian, Jonathan Reuter, Raluca Roman (discussant), Nikolai Roussanov, Ali Shourideh, Jerome Taillard, Mathieu Taschereau-Dumouchel, Luke Taylor, Nicolas Serrano-Velarde (discussant), Amir Yaron, Bilge Yilmaz, John Zhu, Jeffrey Zwiebel, participants at the 9th Annual Corporate Finance Conference at the Washington University in St. Louis, the UNC's Roundtable for Junior Faculty in Finance, the 2013 EFA Meeting, the Fourth FARFE conference, the 2013 NFA Meeting, the 2014 AFA Meeting, the 2014 Financial Intermediation Research Society Conference, and seminar participants at Bentley University, Boston College, University of California, Los Angeles, University of Mannheim, University of Massachusetts Amherst, and University of Pennsylvania for helpful comments and discussions. Email: dlevit@wharton.upenn.edu, nadya.malenko@bc.edu.

Introduction

Why do corporate boards look the way they are? Are boards structured optimally to maximize shareholder value, and how do board regulations affect their composition? To a large extent, the structure of corporate boards is governed by the labor market for directors. On the demand side, firms decide which directors to invite based on directors' reputation and on the preferences of those controlling the nomination process. On the supply side, directors seek to develop their reputation in order to gain more board seats and thereby obtain prestige, power, compensation, and access to valuable networks. Thus, directors' reputation plays an important role, affecting both directors' actions and the structure of corporate boards.

A number of recent institutional and regulatory changes to the director selection process have affected the labor market for directors and the value of reputation. Examples include a shift from plurality to majority voting, proxy access proposals, restrictions on the number of directorships, and increased boardroom transparency. These rules and practices also vary substantially across countries. However, the effect of these factors is not well understood, and some of the recent changes are highly debated.[1] This paper sheds light on these issues by developing a theory of the labor market for directors and studying how directors' reputational concerns affect board structure, directors' behavior, and ultimately shareholder value.

Our key observation is that directors care about two conflicting types of reputation, and which type of reputation is rewarded more in the labor market depends on the aggregate quality of corporate governance. If governance is strong and boards of other firms protect the interests of their shareholders, then building a reputation for being shareholder-friendly can help in obtaining more directorships. Conversely, if governance is weak and boards of other firms are captured by their managers, who want to maintain power, then having a management-friendly reputation can be more useful in getting additional board seats. The empirical evidence is

[1] See, for example, "The Proxy Access Debate," *New York Times*, October 9, 2009.

consistent with the importance of both types of reputation. Some papers, such as Coles and Hoi (2003) and Fich and Shivdasani (2007), find that directors who demonstrate shareholder-friendly behavior and monitor the management are more likely to gain additional directorships. Others, such as Helland (2006) and Marshall (2011), find that shareholder-friendly actions actually hurt directors' chances of being invited to other boards. Moreover, Zajac and Westphal (1996), Eminet and Guedri (2010), and Bouwman (2011) find evidence that is directly consistent with the existence of conflicting reputational concerns: firms controlled by shareholders (managers) are more likely to invite directors who have demonstrated shareholder-friendliness (management-friendliness) in their previous board positions.[2]

To study how these conflicting reputational concerns affect directors' behavior and firm value, we develop a model with three key components. First, being a board member allows a director to affect corporate governance in his firm and thereby change the allocation of control between management and shareholders. Second, whether a director is shareholder-friendly or management-friendly is the director's private information, and by allocating control to either managers or shareholders, directors can affect the market's perception of their shareholder-friendliness. Third, the allocation of control in a given firm determines, among other things, which type of directors it is looking for. In particular, firms that are controlled by shareholders (management) have a demand for shareholder-friendly (management-friendly) directors. Therefore, the aggregate quality of corporate governance, the structure of boards, and the type of reputation that is more valuable in the labor market, are all determined in equilibrium.

We show that directors' reputational concerns lead to strategic complementarity of corporate governance across firms. In particular, stronger governance in one firm leads to stronger governance in its peer firms, and vice versa.[3] Intuitively, when most other firms have weak

[2] Section 3 provides a review of these and other relevant papers in the empirical literature. See also Adams, Hermalin and Weisbach (2010) for a discussion of this reputational trade-off.

[3] In what follows, we use the term "strong corporate governance" for firms in which shareholders have control, and "weak corporate governance" for firms in which the management has control.

corporate governance, the decision of whom to invite to the boards of these firms is controlled by managers. Thus, to increase their chances of obtaining additional directorships, directors have incentives to build a reputation for being management-friendly. This type of reputation can be established by giving more control to the managers of their firms and not interfering with their decisions, leading to weaker governance. Conversely, when most other firms have strong corporate governance, directors will strengthen corporate governance of their firms to build a reputation for being shareholder-friendly.

Our paper thus identifies a novel channel of strategic complementarities between firms, which works through directors' reputational concerns in the labor market. Strategic complementarities arise due to the dual role that directors' actions have on the supply and demand in the market for directors: in addition to affecting directors' own reputation (supply), they also affect which type of reputation is more valuable in the market (demand).[4]

Strategic complementarity of governance has two implications. First, due to strategic complementarity, a small regulatory change, such as a marginal increase in the required percentage of independent directors, can have a very significant effect on the aggregate quality of governance. Second, strategic complementarity implies that there can be multiple equilibria, characterized by the aggregate quality of corporate governance. In particular, we show that when directors' reputational concerns are sufficiently important, an equilibrium in which aggregate governance is strong and the labor market rewards directors for being shareholder-friendly co-exists with a weak governance equilibrium, in which a management-friendly reputation is more valuable. In this respect, the strength of corporate governance is self-fulfilling, and hence, countries and industries with similar characteristics can have very different governance systems.

Our analysis demonstrates that the effect of various corporate governance polices crucially

[4]The existing literature on labor markets points out other channels of strategic complementarities between firms: strategic complementarities can arise when both workers and firms make their investment or entry decisions and there are search frictions (e.g., Acemoglu (1996), Laing, Palivos and Wang (1995)), or when there are increasing returns to scale in either the matching technology (e.g., Diamond (1982)) or the production function (e.g., Benhabib and Farmer (1994)).

depends on the existing aggregate quality of corporate governance. Consider a policy that strengthens directors' reputational concerns, such as increasing the maximum allowed number of directorships a single individual can hold.[5] We show that when directors become more concerned about their reputation in the labor market, governance becomes even stronger in systems with strong governance, where a shareholder-friendly reputation is more valuable. However, in systems where managers are in control and directors are rewarded more for being management-friendly, stronger reputational concerns weaken governance even further. In other words, directors' reputational concerns amplify the existing aggregate quality of corporate governance. This result suggests that restrictions on the number of board seats a single director can hold are more likely to be beneficial in countries with weak governance systems.

For a similar reason, policies that indirectly affect directors' reputational concerns can also be a double-edged sword, whose effect depends on the existing governance system. In particular, we show that increasing board size or improving boardroom transparency is likely to strengthen corporate governance if aggregate governance is already strong, but weaken governance even further if aggregate governance is weak. Thus, our study highlights that due to externalities in the labor market for directors, board size affects governance not only within but also across firms, and that improved transparency may have adverse consequences.[6]

Finally, we show that due to directors' reputational concerns, policies that strengthen corporate governance may actually decrease shareholder welfare for two distinct reasons. First,

[5] Directors' reputational concerns can also be affected by regulations that change the value of a given directorship or include term or age limits on directors. Note that restrictions on the number of directorships are often introduced to allow directors to devote more time and attention to the firms on whose boards they serve. We abstract from director busyness and highlight a novel effect of restrictions on the number of directorships, working through directors' reputational concerns.

[6] One policy that has increased boardroom transparency is the 2004 Securities and Exchange Commission (SEC) disclosure law, which requires companies to disclose if one of their directors leaves the board due to a disagreement. Prior to the SEC ruling, disclosure was only required if the director leaving the firm requested his resignation letter to be made public. The new ruling requires all such departures to be disclosed in the firm's 8-K filing within four business days after the event, even if the director did not provide any written correspondence or request that the matter is made public. In China, a somewhat similar 2004 law requires firms to disclose the names of those independent directors who vote in dissent (Jiang, Wan, and Zhao (2013)).

when aggregate governance becomes stronger and hence a shareholder-friendly reputation becomes more valuable, directors will allocate more control to shareholders solely in order to signal their shareholder-friendliness, and not because more shareholder control is actually optimal. This may lead to inefficiently high levels of shareholder control in cases when leaving some control to the management is valuable due to management expertise or the importance of managerial initiative. Second, strong governance can make a shareholder-friendly reputation so valuable that even the most management-friendly directors will take observable actions that will help them to be perceived as shareholder-friendly. This makes it difficult for shareholders to understand directors' intrinsic characteristics and hence to make informed director appointment decisions.[7]

The paper proceeds as follows. The remainder of the section discusses the related literature. Section 1 introduces the setup. Section 2 presents the analysis, including the comparative statics results, implications for welfare, and the discussion of several extensions. Section 3 offers testable predictions and describes the related empirical literature. Section 4 concludes and discusses other potential applications of our framework. All proofs are delegated to the Appendix and supplemental results are given in the Online Appendix.

Related literature

Our paper is related to the literature on reputational concerns, where agents distort their decisions to convince the market that their quality is high (e.g., Holmstrom (1999)). As in our paper, reputational concerns in these models can lead to strategic interactions between agents or between the agent and the market.[8] In contrast to most of the existing literature,

[7] We also show that an increase in the proportion of independent directors on the board, which is a policy intended to benefit shareholders, may actually lead to a lower probability of shareholder control.

[8] For example, reputational concerns can create herding behavior (e.g., Scharfstein and Stein (1990) and Zwiebel (1995)). Relatedly, Ordonez (2013) shows that reputational concerns in credit markets lead to strategic complementarities in risk-taking between borrowers.

our model features two conflicting types of reputation - shareholder-friendly and management-friendly. In Bar-Isaac and Deb (2013) and Bouvard and Levy (2013), the agent also cares about reputation with two different audiences (the two audiences in our setting are shareholders and managers), but the unique feature of our model is that the actions a player takes to build a certain type of reputation increase the value of this reputation for other players and, due to strategic complementarities, the equilibrium market value of this reputation.[9]

In the context of directors' reputational concerns, our paper is related to Song and Thakor (2006), Ruiz-Verdu and Singh (2011), and Levit (2012). While these papers focus on board-management interactions within a single firm, we study how directors' reputational concerns affect all firms in the economy and emphasize the externalities in corporate governance.

The literature has pointed out that governance externalities can arise from competition for managers (Acharya and Volpin (2010), Acharya, Gabarro, and Volpin (2013), and Dicks (2012)), the takeover market (Burkart and Raff (2013)), and the quality of reported earnings (Nielsen (2006) and Cheng (2011)). Our paper identifies a novel channel of governance externalities working through directors' reputational concerns. To our knowledge, this is the first paper to model the labor market for directors and its effect on equilibrium board structures. In this respect, our paper contributes to the literature that studies how the structure of the board affects board decisions.[10]

[9] See also Austen-Smith and Fryer (2005), Frenkel (2014), and Gertner, Gibbons, and Scharfstein (1988). Dewatripont, Jewitt, and Tirole (1999) study a model of career concerns with multi-tasking and show that if the agent has incentives to demonstrate high talent and talent and effort are complements, multiple equilibria can exist. Because their paper does not feature conflicting types of reputation, the reason for multiplicity of equilibria in their model is different from ours.

[10] See, e.g., Adams and Ferreira (2007), Chakraborty and Yilmaz (2011), Harris and Raviv (2008), Hermalin and Weisbach (1998), Levit (2012), Malenko (2014), and Warther (1998).

1 Setup

There are two identical firms in the economy, and the board of each firm consists of $K \geq 2$ directors.[11] The game has two stages - the allocation of control stage, followed by the director labor market stage.

At the first stage, each director decides whether to vote for a proposal that transfers control of the firm from the manager, who has control by default, to the shareholders. For example, the director can push for the separation of the CEO and board chairman positions, for a higher proportion of independent directors on the board, or for board declassification. Other shareholder-friendly actions include adopting majority voting for director elections, providing proxy access to shareholders, and implementing nonbinding shareholder proposals.

The voting decisions are binary and are made simultaneously by all directors in both firms. Let $e_{ik} \in \{0,1\}$ be the voting decision of director k in firm i. If $e_{ik} = 1$, the director votes in favor of the proposal to shift control to shareholders. If $e_{ik} = 0$, the director votes against the proposal. Let $\chi_i \in \{0,1\}$ be the variable that captures who has control of firm i after the vote, where $\chi_i = 1$ stands for shareholder control, and $\chi_i = 0$ stands for management control. The collective decision-making rule is as follows: if at least $T \in \{1,...,K\}$ directors of firm i vote in favor of the proposal, then shareholders of firm i obtain control, and otherwise, management retains control. In other words, $\chi_i = 1$ if and only if $\sum_{k=1}^{K} e_{ik} \geq T$. For example, if K is an odd number and $T = \frac{K+1}{2}$, then the collective decision-making rule is a simple majority requirement. We assume that individual votes are not observable, but the allocation of control $\chi = (\chi_i, \chi_j)$ is observable.[12]

We assume that directors differ in their shareholder-friendliness. A shareholder-friendly director has a higher relative benefit from shareholder control than a management-friendly

[11] In Section 2.3.4, we discuss an extension to a general number of firms. In unreported analysis, we analyze the case $K = 1$ and the case of asymmetric firms and obtain similar results.

[12] In Section 2.3.3, we discuss an extension in which individual directors' votes are observable as well.

director. This heterogeneity could be due to different objectives or differences in opinion. For example, even if all directors aim to maximize shareholder value, they may disagree on whether the best way to achieve this objective is by giving control to shareholders or to the manager.[13] In particular, the type of director k of firm i is θ_{ik}, where θ_{ik} is distributed according to a continuous symmetric distribution function $F(\theta)$ with mean $\mathbb{E}[\theta]$, bounded density $f(\theta)$, and full support on $\mathbb{R}$.[14] The direct utility of a director of type θ_{ik} from the allocation of control in firm i is $v(\chi_i, \theta_{ik})$, where $v(1, \cdot)$ and $v(0, \cdot)$ are non-negative and continuously differentiable. We assume that $v(1, \theta)$ is increasing in θ and $v(0, \theta)$ is decreasing in θ. Thus, high θ stands for shareholder-friendliness, and low θ stands for management-friendliness. Types are independent across directors, and the type of each director is the director's private information.[15]

It is useful to define a director's relative benefit from shareholder control:

$$\Delta(\theta) \equiv v(1, \theta) - v(0, \theta). \tag{1}$$

Given our assumptions on $v(1, \cdot)$ and $v(0, \cdot)$, it follows that $\frac{\partial \Delta(\theta)}{\partial \theta} > 0$. We make the following additional assumptions on $\Delta(\theta)$. First, we assume that $\lim_{\theta \to \infty} \Delta(\theta) = \infty$ and $\lim_{\theta \to -\infty} \Delta(\theta) = -\infty$. This assumption ensures that if a director is sufficiently shareholder-friendly (management-friendly), his direct utility from shareholder (management) control is sufficiently large and induces him to vote in favor (against) shareholder control despite any

[13] Giving control to the manager could enhance shareholder value either because the success of the firm depends on managerial initiative and firm-specific investments that the manager has incentives to take only if he has control (e.g., Grossman and Hart (1986) and Hart and Moore (1990)) or because the manager has expertise and private information that he will not communicate to the board unless he has control (e.g., Adams and Ferreira (2007) and Harris and Raviv (2008)). In Section 2.3.1, we build on this interpretation and discuss the case in which the relative value from shareholder control can differ across firms.

[14] All the results hold for asymmetric distributions of types as well.

[15] It is not necessary for our results that θ_{ik} is perfectly transferable across firms. Directors may adapt their behavior and conform to the existing corporate governance system of the firms in which they serve as board members. As long as there is some level of persistence of directors' types, our results continue to hold. Consistent with the assumption that θ_{ik} is transferable across firms, Bouwman (2011) finds that a firm's governance practices move in the direction of governance practices of other firms its directors serve at.

reputational concerns. This assumption is made for simplicity and does not affect the main results.[16] Second, we assume $\mathbb{E}[\Delta(\theta)] = 0$. This means that on average, directors' relative benefit from shareholder control is zero.[17] Finally, we assume that directors do not incur any costs of voting against management. If such costs were present, directors would have weaker incentives to vote for shareholder control, especially due to free-riding within the board. Since our focus is on externalities between firms, we abstract from costly voting and free-riding, but our main results would continue to hold in the setting with costly voting as well.

At the second stage, each firm can be hit by a shock, in which case exactly one of its directors resigns and the firm has to appoint a new director. For example, directors may have to resign due to health issues, family reasons, retirement, or because they have been appointed to an executive position. The shocks are independent from the allocations of control χ_i, from directors' types θ_{ik}, and are independent across firms. The probability that the firm is hit by a shock is $\delta \in (0,1)$, and each of K directors has an equal chance of being hit by the shock. Thus, for each director, the unconditional probability of resigning is $\frac{\delta}{K}$. Directors get utility from the allocation of control in their firm whether or not they resign. If a director resigns from the board, he no longer participates in the labor market for directors and does not get any direct utility from resignation. If a director does not resign, he can also be appointed to the board of the other firm if that firm was hit by a resignation shock and needs a new director.

If a director resigns, his firm searches for a new director. The firm can either hire any director of its peer firm who did not resign, or an outside candidate, who is not serving on any board. We assume that the supply of outside candidates is unlimited and that outside candidates are drawn from the same distribution F, so their expected type equals $\mathbb{E}[\theta]$.

If a director is hired by another firm, he gets an additional utility. Specifically, if a director

[16] If $\Delta(\theta)$ is bounded, then in addition to the threshold equilibria described below, there also exist pooling equilibria, in which all types follow the same strategy of voting in favor (or against) shareholder control, but the results do not change qualitatively.

[17] All results, except the analysis of director welfare in the Online Appendix, hold for any value of $\mathbb{E}[\Delta(\theta)]$, including infinity. In the Online Appendix, we discuss how the welfare analysis depends on the sign of $\mathbb{E}[\Delta(\theta)]$.

from firm i joins the board of firm j, he gets $\alpha > 0$. The parameter α can be thought of as the strength of directors' reputational concerns.[18] While directors' financial compensation might be affected by the demand and supply of directors in the labor market, a large component of directors' utility from board seats is non-pecuniary. Indeed, when asked about their personal benefits from serving on the board, directors list prestige, valuable connections, intellectual stimulation, power, and the opportunity to develop new areas of expertise as being more important than financial compensation (see, e.g., Lorsch and MacIver (1989) and the PwC's 2013 Annual Corporate Directors Survey). Matveyev (2013) notes that all outside directors within a firm usually get the same pay, and almost all within-firm variation in compensation comes from the fees directors get from serving on special committees. For these reasons, and for simplicity, we abstract from the effects of the labor market on α and take it as a given parameter. In the Online Appendix, we endogenize α by assuming that the controlling party can offer the new director a contract that is contingent on his actions after he joins the firm. In addition, in unreported analysis, we show that all results of the basic model continue to hold even if α depends on the shareholder-friendliness of the director and the allocation of control in the firm he is joining.[19]

Importantly, the allocation of control in the firm affects who makes the new director appointment decision if one of the directors resigns. Specifically, if the manager has control ($\chi_i = 0$), then the manager makes the appointment decision, and if shareholders have control ($\chi_i = 1$), then shareholders make the appointment decision.[20] We assume, as described in detail below,

[18] We also assume that an outside candidate gets utility α if he joins the board of firm i or j, and utility zero otherwise. This assumption implies that the aggregate utility of directors from the labor market does not depend on whether the vacancies are filled with incumbent directors or outside candidates. All the results of the paper will continue to hold if these assumptions are relaxed. The only exception is Lemma 8 in the Online Appendix, which analyzes the aggregate utility of directors.

[19] Specifically, we assume that if a director of type θ joins a firm with control χ, he gets utility $\alpha(\chi, \theta)$, where $\alpha(1, \theta)$ increases and $\alpha(0, \theta)$ decreases in θ. This captures the idea that a shareholder-friendly director values an appointment to a shareholder-controlled firm more than a management-friendly director, and vice versa.

[20] In practice, the extent to which shareholders have control over director elections depends on a number of governance characteristics that vary across firms. First, it depends on how easy it is for shareholders to nominate their own candidates, in particular, on whether the board is staggered and whether the firm has granted proxy

that shareholders have a preference for more shareholder-friendly directors and the manager has a preference for more management-friendly directors. Denote by $h_i\left(\chi_i,\chi_j\right)\in\{0,1\}$ the hiring decision of firm i based on the allocation of control in both firms and given that one of the directors of firm i has resigned. Specifically, if the firm hires an outside candidate, then $h_i=0$, and if it hires one of the directors of firm j, then $h_i=1$. The hiring decision of firm i depends on the allocation of control in firm j since the allocation of control in a firm is informative about the shareholder-friendliness of its directors. In particular, let π_{jk} denote the reputation of director k in firm j, defined as the expected type of the director at the beginning of the second stage. Since directors' individual votes are not observable, all directors within a firm will have the same reputation in equilibrium. The reputation of directors in firm j will be a function of the allocation of control χ_j and will be endogenously determined by the voting strategies $e_{jk}\left(\theta_{jk}\right)$, $k=1,...,K$. We denote by $\pi_j\left(\chi_j\right)=\mathbb{E}\left[\theta_{jk}|\chi_j\right]$ the equilibrium expected type of directors in firm j given the allocation of control χ_j. We assume that if several directors of firm j have the same reputation and did not resign, they are equally likely to be invited to firm i. We also assume that firms prefer hiring a director currently serving on the other board over hiring an outside candidate whenever the two directors have the same reputation.

Shareholders and managers derive direct utility from the allocation of control in their firm, as well as utility from the composition of their board. We assume that shareholders prefer shareholder control over management control and a shareholder-friendly board over a management-friendly board and that management has the opposite preferences. We also assume that the aggregate shareholders' and managers' utility is higher under shareholder control and with a more shareholder-friendly board, reflecting the idea that management control can create inefficiencies. Specifically, if after the second stage the profile of directors' types in firm i is $\left(\theta_{i1},...,\theta_{iK}\right)$ and $\bar{\theta}_i\equiv\frac{1}{K}\sum_{k=1}^{K}\theta_{ik}$ is the board's average type, then the utility of shareholders

access to large shareholders. Second, it depends on whether the firm uses majority or plurality voting for director elections. Finally, it is affected by the overall independence of the board: the more independent the board is, the easier it is for directors to recommend candidates who are not supported by the manager.

and managers is, respectively, given by

$$
\begin{aligned}
u_{SH}\left(\chi_i, \theta_{i1}, ..., \theta_{iK}\right) &= v_{SH}\left(\chi_i\right) + g_{SH}\left(\bar{\theta}_i\right), \\
u_M\left(\chi_i, \theta_{i1}, ..., \theta_{iK}\right) &= v_M\left(\chi_i\right) + g_M\left(\bar{\theta}_i\right),
\end{aligned}
\tag{2}
$$

where $v_{SH}(0) = v_M(1) = 0$, $v_{SH}(1) \geq v_M(0) > 0$, g_{SH} is an increasing function, g_M is a decreasing function, and $g_{SH} + g_M$ is an increasing function.[21] For tractability, we assume that g_{SH} and g_M are linear. All the results, except the analysis of welfare in Section 2.2, are derived for general increasing functions g_{SH} and g_M.

Note that we implicitly assume that the controlling party only cares about the new director's shareholder-friendliness and abstract from the effect of other relevant factors, such as directors' experience and expertise. Our results will continue to hold in a setting where the party making the appointment decisions also cares about directors' expertise, as long as expertise and shareholder-friendliness are not perfectly correlated across directors. In addition, it is often argued that given the board's dual role as both a monitor and advisor, even shareholders may prefer to have some management-friendly directors on the board (e.g., Adams and Ferreira (2007) and Harris and Raviv (2008)). We discuss this possibility in Section 2.3.1.

Solution concept

Our solution concept is Perfect Bayesian Equilibrium (PBE):

Definition 1 *A Perfect Bayesian Equilibrium is a set of directors' voting strategies $e^*_{ik}(\theta_{ik})$, $e^*_{jk}(\theta_{jk})$, beliefs about directors' types $\pi^*_i(\chi_i)$, and firms' hiring strategies $h^*_i\left(\chi_i, \chi_j\right)$ such that the following conditions are satisfied:*

1. *The voting decision of director k of firm i maximizes his expected utility, where beliefs*

[21] The assumption $v_{SH}(0) = v_M(1) = 0$ is just a normalization.

about directors' types $\pi_i^*(\chi_i)$*, other directors' voting strategies* $e_{ik}^*(\theta_{ik})$*,* $e_{jk}^*(\theta_{jk})$*, and firm* j*'s hiring strategy* $h_j^*(\chi_j, \chi_i)$ *are taken as given.*

2. *The hiring decision of the controlling party of firm* i *maximizes its expected utility, where beliefs* $\pi_j^*(\chi_j)$ *and directors' voting strategies* $e_{ik}^*(\theta_{ik})$*,* $e_{jk}^*(\theta_{jk})$ *are taken as given.*

3. *Whenever possible, beliefs about directors' types are consistent with Bayes' rule, where directors' voting strategies* $e_{ik}^*(\theta_{ik})$*,* $e_{jk}^*(\theta_{jk})$ *are taken as given.*

We restrict attention to equilibria that survive small perturbations in the equilibrium strategies of other directors at the same firm. Formally, we introduce a refinement that is similar to the "trembling hand" refinement in normal form games (e.g., Kreps (1990)).

Definition 2 *Consider any equilibrium in pure strategies. The equilibrium is called trembling hand perfect if for any sequence* $\{\sigma_n\}$*,* $\sigma_n \in (0,1)$*,* $\lim_{n\to\infty} \sigma_n = 1$*, for each firm* i*, director* k_0*, and type* θ*, there exists* $n_0 < \infty$ *such that if:*

(i) all directors of firm i *except* k_0 *play their equilibrium strategy* $e_{ik}^*(\cdot)$ *with probability* σ_n *and make a mistake playing* $1 - e_{ik}^*(\cdot)$ *with probability* $1 - \sigma_n$*, and*

(ii) all directors of firm $j \neq i$ *play their equilibrium strategy* $e_{jk}^*(\cdot)$*,*

then for any $n > n_0$*, given beliefs* $\pi_i^*(1)$ *and* $\pi_i^*(0)$*, the best response of director* k_0 *of firm* i *if his type is* θ *is his equilibrium strategy* $e_{ik_0}^*(\theta)$*.*

2 Analysis

Consider the set of Perfect Bayesian Equilibria of the game that satisfy Definition 2. Consider director k in firm i and his utility from having a reputation π_i. Firm j will search for a new director with probability δ. In this case, if $\chi_j = 1$, i.e., if shareholders of firm j have control, then firm j will hire one of the directors of firm i if and only if their reputation is above the reputation of the outside candidate $\mathbb{E}[\theta]$. Indeed, since g_{SH} is an increasing function of the shareholder-friendliness of the board, shareholders will hire the director with the most shareholder-friendly reputation. Similarly, if $\chi_j = 0$, i.e., the manager of firm j retains control, then firm j will hire one of the directors of firm i if and only if their reputation is below the reputation of the outside candidate. In other words, $h_j\left(\chi_j, \chi_i\right) = 1$ if and only if $\chi_j = 1$ and $\pi_i \geq \mathbb{E}[\theta]$, or $\chi_j = 0$ and $\pi_i \leq \mathbb{E}[\theta]$. Because all directors of firm i are treated symmetrically, conditional on firm j hiring one of the directors of firm i, director k will be hired with probability $\frac{1}{K}$. Hence, if τ_j denotes the ex-ante probability that shareholders of firm j obtain control, then the expected benefit of the director from obtaining reputation π_i is given by $\alpha\frac{\delta}{K}\Gamma(\pi_i, \tau_j)$, where

$$\Gamma(\pi_i, \tau_j) = \tau_j \times 1\{\pi_i \geq \mathbb{E}[\theta]\} + (1 - \tau_j) \times 1\{\pi_i \leq \mathbb{E}[\theta]\}. \tag{3}$$

Note that for any $\underline{\pi}, \overline{\pi}$ such that $\overline{\pi} > \mathbb{E}[\theta] > \underline{\pi}$, $\Gamma(\overline{\pi}, \tau) > \Gamma(\underline{\pi}, \tau)$ if and only if $\tau > 0.5$. Intuitively, whether a director wants to have a shareholder-friendly or a management-friendly reputation depends on the allocation of control in other firms. If managers (shareholders) are the main decision-makers in other firms, i.e., τ is small (large), then the director is more likely to be invited to other boards if he is known for being management-friendly (shareholder-friendly).

Let $U_{ik}(\theta_{ik}, \chi_i)$ be the expected utility of director k in firm i given his type θ_{ik}, allocation of control in his firm χ_i, and taking as given beliefs π_i and the probability of shareholder control in the other firm τ_j. Then $U_{ik}(\theta_{ik}, \chi_i)$ is given by

$$U_{ik}(\theta_{ik}, \chi_i) = v(\chi_i, \theta_{ik}) + \alpha\frac{\delta}{K}\Gamma(\pi_i(\chi_i), \tau_j). \tag{4}$$

Using (4), the following lemma shows that in equilibrium, all directors within a firm follow the same strategy and vote for the proposal if and only if their preference for shareholder control is sufficiently strong.

Lemma 1 *In equilibrium, there exists a finite* θ_i^* *such that* $e_{ik}(\theta) = 1$ *if and only if* $\theta > \theta_i^*$.

Lemma 1 implies that all directors in a firm follow the same threshold voting strategy. The threshold θ_i^* affects the likelihood that shareholders obtain control, which is given by

$$\tau(\theta_i^*) = \sum_{t=T}^{K} C_t^K (1 - F(\theta_i^*))^t F(\theta_i^*)^{K-t}, \tag{5}$$

where $C_t^K = \frac{K!}{t!(K-t)!}$ is the Binomial coefficient. In addition, the threshold θ_i^* affects the formation of directors' reputation. To capture this, we denote the reputation function in equilibrium with a threshold θ_i^* by $\pi_i(\chi_i; \theta_i^*)$. The proof of Lemma 2 derives the expressions for $\pi_i(\chi_i; \theta_i^*)$ and shows that directors whose firm is controlled by shareholders (managers) are perceived to be more (less) shareholder-friendly than an outside candidate:

Lemma 2 *Consider any equilibrium characterized by a threshold* θ_i^*. *Then,*

$$\pi_i(0; \theta_i^*) < \mathbb{E}[\theta] < \pi_i(1; \theta_i^*). \tag{6}$$

Consider the best response function $\beta_i(\theta_j^*)$ of directors in firm i, taking as given that directors in firm j vote for shareholder control when their type exceeds the threshold θ_j^*. The best response function defines the threshold $\beta_i(\theta_j^*)$ such that only types $\theta > \beta_i(\theta_j^*)$ vote for shareholder control. From (4) and the proof of Lemma 1, it follows that $\beta_i(\theta_j^*) = \beta(\theta_j^*)$, where

$$\beta(\theta) \equiv \Delta^{-1}\left(\alpha \frac{\delta}{K}(1 - 2\tau(\theta))\right) \tag{7}$$

and $\Delta^{-1}(\cdot)$ is the inverse of the function $\Delta(\theta)$. Because $\Delta(\theta)$ is strictly increasing, continuous, and takes all values on $(-\infty, +\infty)$, its inverse $\Delta^{-1}(\cdot)$ is a well defined, strictly increasing, and continuous function. Since, as shown in the proof of Lemma 1, $\tau(\theta)$ decreases with θ and takes all values between 0 and 1, the best response function $\beta(\theta)$ increases in θ and takes all values in the interval $\left(\Delta^{-1}\left(-\alpha\frac{\delta}{K}\right), \Delta^{-1}\left(\alpha\frac{\delta}{K}\right)\right)$.

Since the best response threshold of directors in firm i is increasing in the threshold of directors in its peer firm j, the game exhibits *strategic complementarity.* Intuitively, if directors of firm j are more likely to vote for the proposal (θ_j^* decreases), then shareholders of firm j are more likely to get control and have the power to appoint directors to their board. Therefore, the relative reward of directors in firm i from building a shareholder-friendly reputation becomes higher. This increases the incentives of directors in firm i to vote for shareholder control, decreasing the threshold θ_i^*.

The following lemma characterizes the set of equilibria using the properties of $\beta(\theta)$ and the symmetry of the best response functions.

Lemma 3 *An equilibrium always exists, and any equilibrium is symmetric.*

Since all equilibria of the game are symmetric, i.e., $\theta_i^* = \theta^*$ for all i, then any equilibrium θ^* is the solution of $\beta(\theta^*) = \theta^*$. It also follows that the reputation functions $\pi_i(\cdot;\cdot)$ are identical across firms. We denote this function by $\pi(\cdot;\cdot)$. Given (6) and the property of $\Gamma(\pi,\tau)$ discussed above, $\Gamma(\pi(1;\theta^*), \tau(\theta^*)) > \Gamma(\pi(0;\theta^*), \tau(\theta^*))$ if and only if $\tau(\theta^*) > 0.5$. In other words, a shareholder-friendly reputation generates a higher payoff than a management-friendly reputation if and only if there is a higher than 50% chance that the other firm will be controlled by shareholders. As we explain below, identifying which type of reputation is more valuable has important implications for the analysis. Motivated by this argument, the next definition classifies potential equilibria into two types.

Definition 3 *An equilibrium is called shareholder-friendly if* $\tau(\theta^*) > 0.5$ *and management-friendly if* $\tau(\theta^*) < 0.5$.

Due to strategic complementarity, our model can have multiple equilibria. Moreover, the next proposition shows that when reputational concerns are sufficiently important, there always exist at least one shareholder-friendly and at least one management-friendly equilibrium. Thus, equilibria with strong and weak governance can co-exist for a given set of parameters, suggesting that countries or industries with similar characteristics can have different corporate governance systems as an equilibrium outcome.

Proposition 1 *There exist* $\bar{\alpha}$ *and* $\underline{\alpha}$, $0 < \underline{\alpha} \leq \bar{\alpha} < \infty$, *such that:*

(i) *If* $\alpha > \bar{\alpha}$, *there exist at least one shareholder-friendly equilibrium and at least one management-friendly equilibrium.*

(ii) *If* $\alpha < \bar{\alpha}$, *all equilibria are of the same type. In particular, all equilibria are management-friendly if* $\Delta\left(\tau^{-1}(0.5)\right) < 0$ *and shareholder-friendly if* $\Delta\left(\tau^{-1}(0.5)\right) > 0$.

(iii) *If* $\alpha < \underline{\alpha}$, *the equilibrium is unique.*

The reason behind Proposition 1 is that strategic complementarity between firms' corporate governance systems arises due to directors' reputational concerns, represented by parameter α. When α increases, reputation becomes more important for directors, and hence strategic complementarity becomes stronger. Therefore, multiple equilibria are more likely to exist when reputational concerns are significant. Figure 1 illustrates this effect by plotting the best response function of directors for $\delta = 0.2$, $K = 9$, $T = 5$, a standard normal distribution of types, and utility functions $v(1,\theta) = e^{0.4\theta}$ and $v(0,\theta) = e^{-0.2\theta} + e^{0.08} - e^{0.02}$.

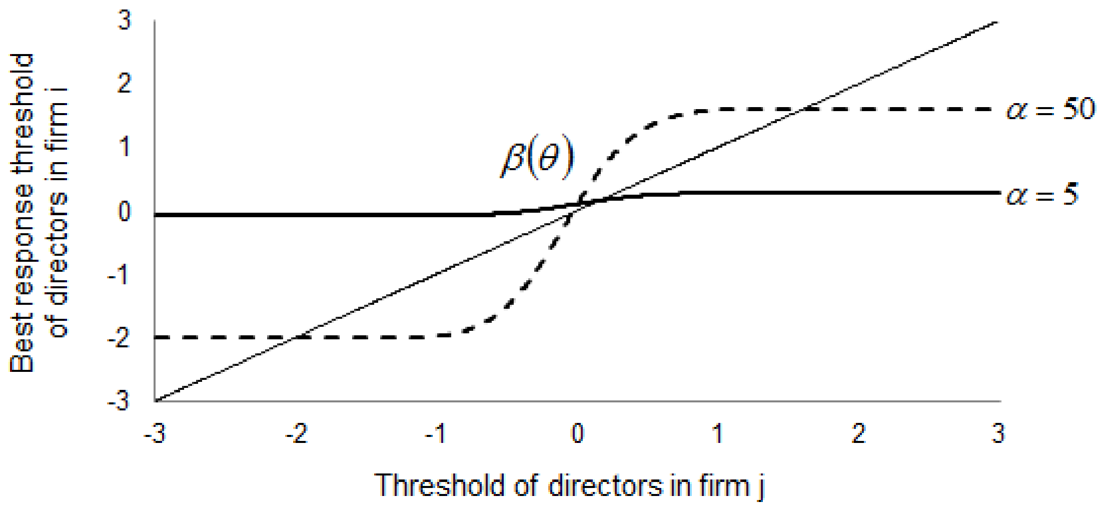

Figure 1: Best response function $\beta(\theta)$ for $\delta = 0.2$, $K = 9$, $T = 5$, $v(1,\theta) = e^{0.4\theta}$, $v(0,\theta) = e^{-0.2\theta} + e^{0.08} - e^{0.02}$, and a standard normal distribution F

First, when $\alpha = 0$, directors do not care about additional board seats and hence make their voting decisions independently of the strategy of directors in the other firm. Hence, the best response $\beta(\theta)$ for $\alpha = 0$ is a constant function with value $\Delta^{-1}(0)$. Therefore, a unique equilibrium exists, and this equilibrium is management-friendly because $\Delta(\tau^{-1}(0.5)) = \Delta(0) < 0$, i.e., the average type prefers management control to shareholder control. When α becomes positive, strategic complementarity arises, and the best response function $\beta(\theta)$ becomes strictly increasing. The solid line in Figure 1 represents $\beta(\theta)$ for $\alpha = 5$. Although $\beta(\theta)$ is increasing, externalities between firms are not strong enough, and hence the game still has a unique, management-friendly, equilibrium (θ^* around 0.16, which corresponds to $\tau(\theta^*)$ around 0.35). However, as Proposition 1 shows, as α increases further, externalities between firms give rise to multiple equilibria, some of which are shareholder-friendly. In particular, when $\alpha = 50$ (the dashed line in Figure 1), the graph of the best response function crosses the

45-degree line in three points, corresponding to three equilibria. Two of them (θ^* around -2.01 and -0.04) are shareholder-friendly, and the third one (θ^* around 1.61) is management-friendly.

2.1 Comparative statics

All equilibria of the game can be ranked by the aggregate quality of corporate governance, defined as the probability that shareholders get control, $\tau(\theta^*)$. Equilibria with a lower θ^* are more shareholder-friendly and feature stronger corporate governance. This section analyzes the comparative statics of corporate governance.

Since the best response function $\beta(\cdot)$ is bounded and increasing, by Tarski's fixed point theorem, $\beta(\cdot)$ has the least and the greatest fixed points (equilibria). We denote these two equilibria by $\underline{\theta}^*$ and $\bar{\theta}^*$ respectively and call them the "most shareholder-friendly" and the "least shareholder-friendly" equilibria of the game.[22] Given the potential multiplicity of equilibria, we focus on the comparative statics in these extremal equilibria, as is common in games of strategic complementarities (e.g., Vives (2005)).[23]

Proposition 2 *Suppose that θ^* is either $\underline{\theta}^*$ or $\bar{\theta}^*$ and let $\tau^* \equiv \tau(\theta^*)$. Then:*

(i) *τ^* increases with α if and only if the equilibrium is shareholder-friendly.*

(ii) *If $F_2(\theta)$ first-order stochastically dominates $F_1(\theta)$, then $\tau_2^* \geq \tau_1^*$.*

(iii) *Given K, τ^* decreases with T.*

(iv) *Suppose $\lambda \equiv \frac{\delta}{K}$ is fixed and consider a change in K:*

(iv.a) *Suppose T is fixed. Then, τ^* increases with K.*

[22] The "most shareholder-friendly" equilibrium can be management-friendly if all equilibria of the game are management-friendly, and vice versa.

[23] For the continuous parameter α, we focus on local comparative statics, when the equilibrium continues to exist upon a small change in the parameter.

(iv.b) Suppose $T = K$ (unanimity rule). Then, τ^* decreases with K.

(iv.c) Suppose K is odd and $T = \frac{K+1}{2}$ (simple majority rule). Let $K_2 > K_1$ and suppose that $\alpha > \bar{\alpha}(K_1)$, where $\bar{\alpha}(K)$ is defined by Proposition 1. Then, $\tau_2^* \geq \tau_1^*$ if and only if the equilibrium is shareholder-friendly.

The first statement of Proposition 2 shows that stronger directors' reputational concerns improve corporate governance only if the equilibrium is shareholder-friendly. This is because in a management-friendly equilibrium, managers of other firms, rather than shareholders, make the appointment decisions, and hence having a shareholder-friendly reputation hurts directors' chances of being invited to other boards. In this sense, directors' reputational concerns amplify corporate governance: as α increases, strong governance systems become stronger and weak systems become weaker. This suggests that a regulation that increases the value of reputation in the director labor market (e.g., increasing the allowed number of directorships a person can hold) strengthens governance only if the existing governance system is sufficiently strong.

According to the second statement of the proposition, if the population of directors becomes more shareholder-friendly (for example, due to a regulation that increases all directors' accountability to shareholders), the equilibrium probability of shareholder control increases. Note also that a higher likelihood of shareholder-friendly directors leads to a higher probability of shareholder control for two reasons. First, keeping directors' threshold strategy θ^* fixed, it is more likely that each individual director's type will be above the threshold. This effect is amplified by the decrease in directors' equilibrium threshold θ^*: knowing that the other firm is now more likely to be controlled by shareholders, each director has stronger incentives to vote for shareholder control and thereby build a shareholder-friendly reputation.

The intuition behind the third statement is straightforward: all else equal, a higher majority requirement reduces the probability that shareholders obtain control. Part (a) of the fourth statement implies that if T is fixed, a larger board size improves corporate governance.

Intuitively, with a larger board, it is easier to deviate from the status quo and transfer control to shareholders. In contrast, part (b) shows that under the unanimity voting rule, a larger board size leads to weaker governance. Indeed, with unanimity, shareholders obtain control only if all directors vote for it. The larger the board, the more likely that at least one director is sufficiently management-friendly and votes against shareholder control. Note that in all parts of the fourth statement, we keep $\lambda \equiv \frac{\delta}{K}$ constant as we increase K, that is, we simultaneously increase δ. This captures the idea that when the board is larger, it is more likely that at least one of its directors will have to resign. If δ remained constant as K increased, then all else equal, each director would be less likely to be hired by the other firm simply because the supply of directors would be larger. This effect is similar to the effect of a decrease in α, which we discuss in part (i) of the proposition. We therefore fix $\frac{\delta}{K}$ to emphasize that K affects the equilibrium in a novel way, which is different from this supply effect.

In practice, boards generally make decisions based on a simple majority rule.[24] Part (c) of the fourth statement shows that under a simple majority rule, a larger board size improves corporate governance if and only if the equilibrium is shareholder-friendly. Thus, increasing board size amplifies governance in the sense that weak governance systems become weaker and strong governance systems become stronger as board size increases. Intuitively, under a simple majority rule, the equilibrium is shareholder-friendly (management-friendly) if the probability that each director votes for shareholder control is greater (smaller) than 0.5. An increase in board size reduces the uncertainty about the outcome of the vote: as K increases, the likelihood that at least half of the board will vote for shareholder control increases (decreases). By making the outcome of the vote in the peer firm more predictable, a larger board size effectively amplifies a director's reputational concerns and thus amplifies corporate governance

[24]For example, the Delaware General Corporation Law states "The vote of the majority of the directors present at a meeting at which a quorum is present shall be the act of the board of directors unless the certificate of incorporation or the bylaws shall require a vote of a greater number." (http://delcode.delaware.gov/title8/c001/sc04/index.shtml)

by a similar intuition as before.

This intuition also implies that corporate governance is more likely to be self-fulfilling when board size is larger. For example, if directors of firm i believe that each director of firm j is likely to vote for shareholders (θ^* is small), a large board size implies that firm j is very likely to be controlled by shareholders, giving directors of firm i strong incentives to vote for shareholders as well. Thus, when K is sufficiently large, beliefs that θ^* is small become self-fulfilling.[25] The following lemma formalizes this intuition.

Lemma 4 *Suppose $\lambda \equiv \frac{\delta}{K}$ is fixed and consider a simple majority rule. If both types of equilibria (shareholder- and management-friendly) co-exist for a given K, they also co-exist for any larger K. Moreover, if $\Delta(\mathbb{E}[\theta]) \neq 0$, there exist α_1 and α_2, $0 < \alpha_1 < \alpha_2 < \infty$, such that:*

(i) If $\alpha \leq \alpha_1$, then only one type of equilibrium exists for any $K \geq 3$.

(ii) If $\alpha \in (\alpha_1, \alpha_2)$, then there exists $\hat{K} > 3$ such that both types of equilibria co-exist if and only if $K \geq \hat{K}$.

(iii) If $\alpha \geq \alpha_2$, then both types of equilibria co-exist for any $K \geq 3$.

We conclude the comparative statics analysis by noting that small changes in parameters are amplified due to strategic complementarity of directors' voting decisions. Consider, for example, a decrease in the voting requirement T. The direct effect of a decrease in T is that if directors' strategies are fixed, control is more likely to be shifted to shareholders since the proposal requires the approval of a smaller number of directors. In addition, realizing that the

[25] This logic also helps explain why statement (*iv.c*) of Proposition 2 might not hold if $\alpha < \bar{\alpha}$: if $\alpha < \bar{\alpha}$, and hence all equilibria are of the same type, the amplification effect of K implies that an increase in K could give rise to an additional equilibrium of a different type (similar to an increase in α in Proposition 1). Finally, it is important to note that $\alpha > \bar{\alpha}$ is a sufficient, but not necessary, condition for (*iv.c*) to hold. For example, (*iv.c*) also holds if α is sufficiently small and the equilibrium is unique both before and after a change in K. Generally, (*iv.c*) holds whenever the type of the extremal equilibrium does not change with a change in K.

peer firm is now more likely to be controlled by shareholders, directors have stronger incentives to create a shareholder-friendly reputation. This second, indirect effect, induces directors to vote for shareholder control and magnifies the direct effect. This amplification effect is standard in games with strategic complementarities.[26]

2.2 Welfare

In this section, we analyze the welfare implications of the model. We start by deriving players' expected utilities as a function of the equilibrium threshold θ^* and then use it to compare equilibria in terms of social welfare and Pareto efficiency.

Consider the expected utility of shareholders and management of any firm i. It consists of the utility from the allocation of control and the utility from the composition of the board. The expected utility of shareholders from the allocation of control in their firm is $\tau(\theta^*) \times v_{SH}(1)$. Similarly, the expected utility of management from the allocation of control in its firm is $(1-\tau(\theta^*)) \times v_M(0)$. If $c(\theta^*)$ denotes the expected composition (average type) of the board after the labor market stage in equilibrium with a threshold θ^*, then, because g_{SH} and g_M are linear, the total expected utility of shareholders of firm i is given by

$$W_{Shareholders}(\theta^*) = \tau(\theta^*) \times v_{SH}(1) + g_{SH}(c(\theta^*)), \tag{8}$$

and the total expected utility of the management is given by

$$W_{Management}(\theta^*) = (1-\tau(\theta^*)) \times v_M(0) + g_M(c(\theta^*)), \tag{9}$$

[26]Formally, the proof of Proposition 2 shows that for a parameter p, $\frac{\partial\theta^*}{\partial p} = M(\theta^*) \times \frac{\partial\beta(\theta)}{\partial p}|_{\theta=\theta^*}$, where $\frac{\partial\beta(\theta)}{\partial p}|_{\theta=\theta^*}$ captures the direct effect of parameter p, and the multiplier $M(\theta^*) = \frac{1}{1-\frac{\partial\beta(\theta)}{\partial\theta}|_{\theta=\theta^*}}$ is greater than 1.

where

$$
\begin{aligned}
c(\theta^*) \quad = \quad & (1-\delta)\,\mathbb{E}[\theta] + \delta\tau(\theta^*)\,(1-\tau(\theta^*))\left(\frac{K-1}{K}\pi(1;\theta^*) + \frac{1}{K}\mathbb{E}[\theta]\right) \\
& +\delta\tau(\theta^*)^2\,\pi(1;\theta^*) + \delta(1-\tau(\theta^*))\,\tau(\theta^*)\left(\frac{K-1}{K}\pi(0;\theta^*) + \frac{1}{K}\mathbb{E}[\theta]\right) \\
& +\delta(1-\tau(\theta^*))^2\,\pi(0;\theta^*)\,.
\end{aligned}
$$

The first term corresponds to the case where there is no resignation shock in firm i, and hence the expected average type is the prior regardless of the allocation of control. All other terms relate to cases where firm i is hit by a resignation shock. The second term corresponds to the case where shareholders get control in firm i but management retains control of firm j. Since shareholders of firm i then hire the outside candidate with reputation $\mathbb{E}[\theta]$ to replace the resigning director, and since the reputation of the remaining directors of firm i is $\pi(1;\theta^*)$, the expected average type of the board conditional on this event is $\frac{K-1}{K}\pi(1;\theta^*) + \frac{1}{K}\mathbb{E}[\theta]$. The third term corresponds to the case where shareholders get control in both firms. Since shareholders then hire a director of the other firm, whose reputation is $\pi(1;\theta^*)$, and since the reputation of the remaining directors is also $\pi(1;\theta^*)$, the expected average type of the board conditional on this event is $\pi(1;\theta^*)$. The fourth and fifth terms are derived similarly and correspond to the cases where firm i is controlled by management, while firm j is controlled by shareholders and management, respectively. The next result describes several properties of $c(\theta^*)$.

Lemma 5 *$c(\theta^*)$ has the following properties:*

(i) If $\tau(\theta^*) \in \left\{0, \frac{1}{2}, 1\right\}$, then $c(\theta^*) = \mathbb{E}[\theta]$;

(ii) If $\tau(\theta^*) \in \left(0, \frac{1}{2}\right)$, then $c(\theta^*) < \mathbb{E}[\theta]$;

(iii) If $\tau(\theta^*) \in \left(\frac{1}{2}, 1\right)$, then $c(\theta^*) > \mathbb{E}[\theta]$.

Since g_{SH} is increasing and g_M is decreasing, Lemma 5, combined with (8) and (9), implies that shareholders' (managers') expected utility in any shareholder-friendly equilibrium ($\tau(\theta^*) > \frac{1}{2}$) is strictly higher (lower) than their expected utility in any management-friendly equilibrium ($\tau(\theta^*) < \frac{1}{2}$). However, it is generally not true that more shareholder-friendly equilibria (with higher $\tau(\theta^*)$) feature higher shareholder value and lower management value.

The intuition for this result is the following. Consider the shareholders of firm i. The direct effect, which we call the control effect, is that as θ^* decreases and hence $\tau(\theta^*)$ increases, shareholders are more likely to get control. Thus, their utility increases both due to direct benefits of control and due to their control over the director appointment decisions, i.e., their ability to invite directors with a shareholder-friendly reputation after learning about their type from their voting decisions. However, the indirect effect is that when the equilibrium is very shareholder-friendly (θ^* is very low), directors of firm j vote for shareholder control almost regardless of their types. As a result, shareholders of firm i learn very little about the type of directors of firm j from these directors' decision to give control to shareholders. Thus, the privilege of controlling the composition of the board is offset by the inability to find directors who are intrinsically shareholder-friendly. For example, when $\tau(\theta^*) \to \{0, 1\}$, there is no learning at all and hence no benefit from controlling the composition of the board. We call this novel effect the learning effect.[27] It is important to note that shareholders always benefit from more shareholder control in their own firm (i.e., from lower θ_i^*). The learning effect only applies to shareholders' utility from the allocation of control in the other firm, θ_j^* : when θ_j^* is low, shareholders of firm i learn little about directors from firm j and may prefer a higher θ_j^*.

Lemma 5 implies that $c(\theta^*)$ is non-monotonic. Figure 2 presents the graph of $c(\theta^*)$ for $\delta = 0.2$, $K = 9$, $T = 5$, and a standard normal distribution of types. Generally, the sign of

[27]Note that $c(\theta^*) = \mathbb{E}[\theta]$ in the case $\tau(\theta^*) = \frac{1}{2}$ as well. However, here, the intuition is different: in contrast to equilibria where $\tau(\theta^*) \to \{0, 1\}$, learning takes place when $\tau(\theta^*) = \frac{1}{2}$. Nevertheless, the expected type of the board equals the prior because the increase in board shareholder-friendliness when shareholders have control is exactly offset by the decrease in shareholder-friendliness when management has control.

$\frac{\partial c(\theta^*)}{\partial \theta^*}$ determines whether the learning effect dominates the control effect. When $\frac{\partial c(\theta^*)}{\partial \theta^*} < 0$, the expected composition of the board becomes more shareholder-friendly as the equilibrium becomes more shareholder-friendly. In these cases, the learning effect is always dominated by the control effect, and hence shareholders' expected utility increases in the shareholder-friendliness of the equilibrium. However, when $\frac{\partial c(\theta^*)}{\partial \theta^*} > 0$, a more shareholder-friendly equilibrium can decrease shareholder welfare because the board becomes more management-friendly.

The presence of the learning effect implies that policies that strengthen corporate governance in the economy may not always benefit shareholders. For example, even though a regulation that increases directors' accountability to shareholders improves governance according to the comparative statics in Proposition 2 (ii), it does not necessarily increase shareholder welfare. Moreover, due to strategic complementarity, even a policy that only targets firm i by improving its corporate governance can nevertheless harm shareholders of firm i. This is because such a policy spills over to other firms, making directors of other firms more likely to give control to shareholders regardless of their type and thus making it difficult for shareholders of firm i to make informed director appointment decisions.

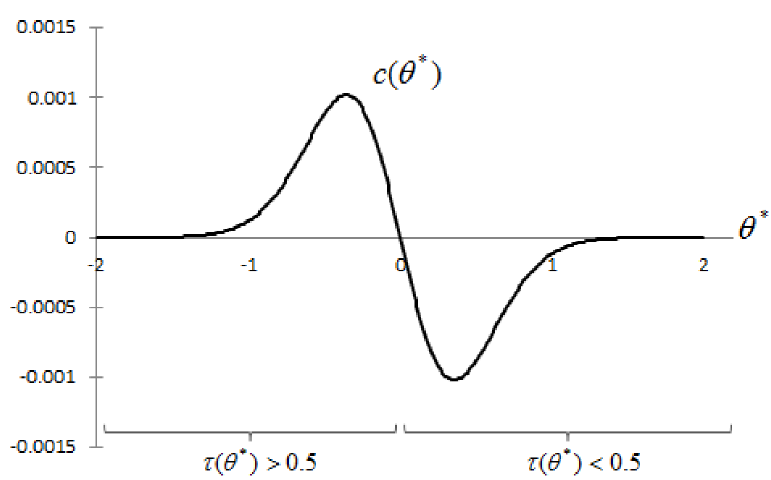

Figure 2: The average type of the board $c(\theta^*)$ for $\delta = 0.2$, $K = 9$, $T = 5$, and a standard normal distribution F

In the Online Appendix, we derive the aggregate expected utility of directors and show that it is non-monotonic in θ^*. We also show that under certain conditions, directors' expected utility increases in the shareholder-friendliness of the equilibrium (decreases in θ^*) if and only if the equilibrium is shareholder-friendly.

2.2.1 Implications for social welfare and Pareto efficiency

Due to the learning effect and the non-monotonicity of the expected utility of directors, the effect of θ^* on the social welfare function that takes into account all players, including shareholders, managers, and directors, is generally ambiguous. Suppose, however, that the social welfare function puts a sufficiently small weight on the welfare of directors, so that the effect of θ^* on social welfare is determined by its effect on the combined utility of shareholders and management only. Then, the assumptions $v_{SH}(1) \geq v_M(0)$ and $g'_{SH} + g'_M \geq 0$, together with Lemma 5, imply that social welfare in any shareholder-friendly equilibrium is higher than social welfare in any management-friendly equilibrium. Moreover, note that

$$\frac{\partial}{\partial \theta^*}\left[W_{Shareholders}(\theta^*) + W_{Management}(\theta^*)\right] = \frac{\partial \tau(\theta^*)}{\partial \theta^*}\left[v_{SH}(1) - v_M(0)\right] + \frac{\partial c(\theta^*)}{\partial \theta^*}\left[g'_{SH} + g'_M\right].$$

Therefore, if the learning effect is dominated by the control effect ($\frac{\partial c(\theta^*)}{\partial \theta^*} \leq 0$), then social welfare locally increases with the shareholder-friendliness of the equilibrium.

While the analysis of social welfare is generally ambiguous, more can be said about Pareto efficiency:

Lemma 6

(*i*) *No shareholder-friendly equilibrium is Pareto dominated by a management-friendly equilibrium, and vice versa.*

(*ii*) *If* $\frac{v_{SH}(1)}{g'_{SH}} = \frac{v_M(0)}{-g'_M}$, *then every equilibrium is Pareto efficient.*

In particular, the second statement of the lemma implies that all equilibria are Pareto efficient if there are no efficiency losses from management control and from a management-friendly board, that is, if $v_{SH}(1) = v_M(0)$ and $g'_{SH} + g'_M = 0$.

2.3 Extensions

In this section, we discuss several extensions of the basic model. The formal setups, results, and proofs for these extensions are provided in the Online Appendix.

2.3.1 Value of shareholder control

If managers have high expertise or if they need to be given incentives to make firm-specific investments, shareholders may be better off delegating control to them.[28] We analyze an extension where the optimal allocation of control varies across firms and show that the extended model exhibits strategic complementarity as well. Moreover, directors' reputational concerns may now give rise to excessive shareholder control: if a shareholder-friendly reputation is rewarded in the labor market, directors may allocate control to shareholders to signal their shareholder-friendliness even though management control is optimal. This new type of inefficiency suggests that regulators and exchanges should exercise caution in imposing corporate governance requirements. Suppose, for example, that a new listing standard increased the minimum percentage of independent directors on the board to 75%, which would result in the optimal level of shareholder control if directors had no reputational concerns. However, realizing that other firms are now more likely to be controlled by shareholders, directors would have stronger incentives to transfer control to shareholders in their own firms to signal their shareholder-friendliness. As a result, the regulation could shift the equilibrium to even higher levels of board independence and an excessively high level of shareholder control.

[28] For example, see Grossman and Hart (1986), Hart and Moore (1990), Adams and Ferreira (2007), and Harris and Raviv (2008).

2.3.2 Board independence

To formally study the effect of board independence, we consider an extension where some directors are insiders and always vote for management control. If the strategies of independent directors were not affected by board structure, a higher number of insiders would increase the likelihood of management control. Interestingly, however, if insiders participate in the labor market for directors, then independent directors can be more likely to vote for shareholders in the presence of insiders. Intuitively, an increase in the number of insiders decreases the supply of incumbent shareholder-friendly directors and increases the supply of incumbent management-friendly directors, which increases the relative value of a shareholder-friendly reputation due to competition for board seats. In the Online Appendix, we show that the overall effect can be such that the presence of insiders leads to a higher probability of shareholder control.

2.3.3 Boardroom transparency

While the board's decision-making process is generally opaque, recent regulations have increased boardroom transparency, making the behavior of individual directors more visible (see footnote 6). To capture this, we analyze an extension where directors' individual votes are observed. We show that transparency makes the most (least) shareholder-friendly equilibrium more (less) shareholder-friendly and thereby amplifies corporate governance. This result is similar in spirit to Proposition 2 (i), which shows that directors' reputational concerns amplify governance. Intuitively, this is because transparency strengthens the link between a director's individual vote and his reputation. If aggregate governance is weak and a management-friendly reputation is more valuable, directors may be more reluctant to oppose the management when they know that their actions will be observed. Thus, increasing boardroom transparency with the goal of strengthening a weak governance system is likely to achieve the opposite outcome.

Similar to the basic model, the extended model features strategic *complementarities* between directors's decisions *across* firms. However, transparency also gives rise to strategic *substitutability* between directors' decisions *within* firms. The reason is that directors within a firm compete for the board seat at the other firm and hence can benefit from differentiating their reputation from each other. It follows that the labor market for directors creates incentives for non-conformity within the boardroom.

2.3.4 Multiple firms

We extend the model to $N \geq 2$ firms and show that our main results continue to hold. Importantly, based on the allocation of control across firms after the first stage, the market is divided into two sets: firms controlled by shareholders search among directors with a shareholder-friendly reputation, and firms controlled by managers search among directors with a management-friendly reputation. Thus, there is governance-related segmentation in the labor market for directors. As the number of firms becomes infinitely large, the externalities due to reputational effects disappear. However, given that the labor market for directors is segmented both by industry and by geographical location (see the discussion in Section 3), we think of N as representing the number of firms in the relevant segment and hence not being very large.

3 Empirical predictions

In this section, we discuss our paper in the context of the existing empirical literature and offer new testable predictions. We are not aware of other theories that have these predictions.

The premise of our paper is that directors trade off two conflicting types of reputation – one for being shareholder-friendly and one for being management-friendly. Consistent with the existence of this trade-off, the literature has found mixed results with respect to whether the labor market rewards directors for imposing discipline on the management. Consistent with the view that a shareholder-friendly reputation is rewarded, several papers find that directors are held accountable for failing to monitor the management.[29] Conversely, consistent with the view that a management-friendly reputation is rewarded, Helland (2006) finds that directors of firms charged with fraud experience an increase in the number of outside directorships, and Marshall (2011) shows that directors who resign from the board over a disagreement experience a loss in board seats over the five year period following the dispute.[30]

Most of the existing literature looks at the aggregate number of board seats gained by directors. In contrast, our paper emphasizes that whether directors' shareholder-friendly actions will be rewarded by invitations to boards of other firms crucially depends on the balance of power at these firms. Formally, the first implication is the following.

Prediction 1 *Directors who demonstrate shareholder-friendliness are more (less) likely to be subsequently appointed to boards of firms with stronger (weaker) corporate governance.*

Shareholder-friendly directors can be identified as those who vote against the management (Jiang, Wan, and Zhao (2013)) or leave the board due to a disagreement (Marshall (2011)). Alternatively, one can look at firms where a director holds a board seat and measure the observable changes in these firms' corporate governance during the director's tenure (e.g., removal of antitakeover defenses or CEO-chairman separation). Zajac and Westphal (1996), Eminet and Guedri (2010), and Bouwman (2011) find evidence consistent with the first prediction. For example, Zajac and Westphal (1996) show that directors on boards that have recently increased the ratio of outside directors, separated the CEO and chairman positions, or decreased executive compensation, have fewer subsequent appointments to firms with low board control

[29] Coles and Hoi (2003) show that directors who rejected the Pennsylvania Senate Bill 1310 antitakeover provisions were three times as likely to gain additional directorships than those who retained the provisions. Fich and Shivdasani (2007) find that following a financial fraud lawsuit, directors are likely to lose board seats at other firms, particularly those with strong governance. See also Harford (2003), Yermack (2004), Srinivasan (2005), Ertimur, Ferri, and Stubben (2010), Jiang, Wan, and Zhao (2013), and Fos and Tsoutsoura (2013).

[30] Relatedly, Ertimur, Ferri, and Maber (2011) find no evidence that directors of firms involved in option backdating incur reputational penalties at other firms.

but more appointments to firms with high board control.[31]

Our paper also emphasizes the existence of corporate governance externalities between firms. While governance externalities can be due to several reasons, the unique feature of our model is that externalities arise due to directors' reputational concerns in the labor market. Thus, another empirical implication, which helps distinguish our mechanism from other potential mechanisms, is the following.

Prediction 2 *A positive exogenous shock to corporate governance of one firm improves corporate governance of other firms, and this spillover effect is greater for firms whose directors have stronger reputational concerns.*

Since governance externalities arise through the labor market for directors, they are likely to be stronger across firms in the same segment of the labor market, such as firms in the same geographic area and firms in the same industry. Indeed, the market for directors is somewhat segmented both by geographic location (e.g., Knyazeva, Knyazeva, and Masulis (2013)) and by industry, since firms look for candidates with relevant expertise and industry knowledge (e.g., Dass et al. (2014)). Thus, the empirical predictions of this section are likely to be stronger if a firm's peer group is defined as firms in related industries or firms in close geographic proximity. In this sense, Prediction 2 is consistent with the evidence in Albuquerque et al. (2014), who show that following a cross-border acquisition, the local industry rivals of the target firm experience improvements in corporate governance. In addition, as the results of Section 2.3.4 demonstrate, the externalities between firms become weaker as the number of firms increases. Hence, the empirical predictions of this section should be the strongest when the relative segment of the market is relatively small, for example, if the firm's industry is

[31] Bouwman (2011) shows that a firm is more likely to select an individual as its director if this individual is a director at firms whose governance practices are similar to the firm's existing governance practices. In the context of French firms, Eminet and Guedri (2010) find that directors who implement governance reforms that increase (decrease) control over management are more likely to be appointed to boards with (without) nominating committees and boards with nominating committees dominated by non-executive (executive) directors.

concentrated.

Our analysis has implications for the transparency of board decision-making. In 2004, the SEC adopted a law requiring firms to publicly disclose if one of their directors leaves the board due to a disagreement. A similar 2004 law in China requires firms to disclose if one of their independent directors votes in dissent. Section 2.3.3 shows that increasing transparency amplifies corporate governance: strong governance systems become stronger and weak governance systems become weaker. Hence:

Prediction 3 *Greater boardroom transparency strengthens (weakens) the firm's corporate governance if corporate governance of other firms is strong (weak).*

The model also has implications for policies limiting the number of board seats a single director can hold. Although US laws do not restrict the number of directorships, US firms have been increasingly adopting such restrictions voluntarily.[32] According to the 2012 Spencer Stuart Board Index, 74% of S&P 500 firms now limit the number of directorships for their board members, compared to only 27% in 2006. Since restricting the number of directorships decreases directors' reputational concerns, Proposition 2 (i) suggests the following prediction:

Prediction 4 *A restriction on the number of directorships that each of the firm's directors can hold strengthens (weakens) the firm's corporate governance if corporate governance of other firms is weak (strong).*

In addition to the maximum allowed number of directorships, directors' reputational concerns can be affected by factors such as age and tenure. While several papers (e.g., Marshall (2011) and Jiang, Wan, and Zhao (2013)) study the effect of directors' age and tenure on the likelihood that they take shareholder-friendly actions, these papers do not look at the interac-

[32] Many European and Asian countries impose a limit on the number of board seats a single director can hold, and this limit is different for different countries. See ECGI corporate governance codes at http://www.ecgi.org/codes/all_codes.php and the White Paper on Corporate Governance in Asia (OECD 2003).

tion between directors' reputational concerns and other firms' governance practices. Our paper emphasizes that directors with stronger reputational concerns will be more likely to act in a shareholder-friendly manner only if corporate governance of peer firms is strong.

Finally, the comparative statics with respect to board size in Proposition 2 (iv.c) leads to the following prediction.[33]

Prediction 5 *An increase in board size strengthens (weakens) the firm's corporate governance if corporate governance of other firms is strong (weak).*

Empirically, a larger board size is associated with several characteristics of weak corporate governance. For example, Yermack (1996) finds that small boards are more likely to have non-CEO chairmen, greater levels of director stock ownership, and receive performance-based director fees, and Fahlenbrach (2009) shows that firms with large boards have weaker shareholder rights, higher levels of CEO compensation, and lower pay-for-performance sensitivity of CEO compensation. The evidence is mixed because these papers also find that larger boards have a higher percentage of independent directors. With this caveat, if one interprets the existing evidence as showing a negative relation between board size and governance, Proposition 2 (iv.c) and prediction 5 suggest that the US economy is in a management-friendly equilibrium. Under this assumption, given Proposition 2 and predictions 3 and 4, we would expect increased boardroom transparency to weaken corporate governance, and restrictions on the number of directorships to strengthen corporate governance.

Predictions 3-5 refer to exogenous changes in governance characteristics such as transparency or board size, for example, due to a new regulation. In this case, these characteristics could change in a way that decreases shareholder value in a given firm. If, however, shareholders have to approve the adoption of new governance practices, then such changes will only be made if they increase shareholder value. Under this assumption and assuming that stronger

[33] Proposition 2 (iv.c) is based on the assumption that the board makes decisions based on a simple majority rule. As discussed in Section 2.1, this assumption is consistent with the observed board practices.

governance increases shareholder welfare, our model predicts that a company will only limit the number of directorships its directors can hold and will only decrease transparency and board size if its peer firms have weak corporate governance.

Note also that the key to the above predictions is the existence of two conflicting types of reputation and the idea that corporate governance at peer firms determines which type of reputation is more valuable. Thus, qualitatively, these predictions do not rely on strategic complementarity of directors' voting decisions. However, due to strategic complementarity, the above effects are significantly amplified.[34]

4 Conclusion

This paper develops a model of the labor market for directors and studies how directors' reputational concerns affect corporate governance, the structure of the board, and shareholder value. Whether directors would like to build a reputation for being shareholder-friendly or management-friendly, is determined in equilibrium and depends on the allocation of control between shareholders and managers in other firms. In particular, the labor market only rewards directors for being shareholder-friendly if corporate governance in most firms is strong.

We show that directors' reputational concerns create corporate governance externalities between firms. Stronger governance in one firm leads to stronger governance in other firms and vice versa, and this spillover effect is stronger when directors' concerns about reputation are stronger. As a result, an equilibrium with strong aggregate governance can co-exist with an equilibrium with weak aggregate governance, suggesting that countries and industries with similar characteristics can have different governance systems. We also show that when directors'

[34] For example, relaxing the restriction on the number of directorships for firm A's directors improves corporate governance of firm A if governance of its peer firms is strong, even if governance at these firms is not affected by the change. However, the strategic complementarity effect implies that improved governance in firm A will, in turn, increase the market value of a shareholder-friendly reputation and thus improve governance at peer firms as well, amplifying the initial effect.

reputation in the labor market becomes more important for them, strong governance systems become stronger but weak systems become even weaker. This implies that the effect of certain regulations, such as restricting the number of board seats an individual can hold or increasing transparency of board decision-making, crucially depends on the existing state of corporate governance. Our analysis provides new empirical predictions about director appointments and peer effects in corporate governance.

While the focus of our paper is on the labor market for corporate directors, our framework can be applied to other settings where an agent's decisions affect both his own reputation and the type of reputation that is valued at his workplace. Examples include the CEO's choice of corporate culture (e.g., the level of employee friendliness), an employee's adoption of a new technology, or an academic's choice of research agenda.

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Appendix

Proof of Lemma 1. Director k in firm i maximizes (4) taking $e_{ik}(\theta_{ik})$, $e_{jk}(\theta_{jk})$, and $\pi_i(\chi_i)$ as given. By voting for the proposal, the director will only change the allocation of control in his firm (will only be pivotal) if exactly $T-1$ other directors vote for the proposal. First, suppose that a director is pivotal with a positive probability. The case where some directors are never pivotal is analyzed in the Online Appendix. Conditional on being pivotal, the director's relative utility from voting for the proposal relative to voting against the proposal is

$$v(1,\theta_{ik}) - v(0,\theta_{ik}) + \frac{\alpha\delta}{K}\left[\Gamma(\pi_i(1),\tau_j) - \Gamma(\pi_i(0),\tau_j)\right]. \tag{10}$$

When the director is not pivotal, his vote does not change his utility, and hence the director's relative utility of voting for the proposal conditional on not being pivotal is zero. Hence, the director votes for the proposal if and only if

$$\Delta(\theta_{ik}) > \frac{\alpha\delta}{K}\left[\Gamma(\pi_i(0),\tau_j) - \Gamma(\pi_i(1),\tau_j)\right], \tag{11}$$

where

$$\begin{aligned}\Gamma(\pi_i(0),\tau_j) - \Gamma(\pi_i(1),\tau_j) \quad &= \quad \tau_j \cdot \left(1\{\pi_i(0) \geq \mathbb{E}[\theta]\} - 1\{\pi_i(1) \geq \mathbb{E}[\theta]\}\right) \\ &\quad + (1-\tau_j) \cdot \left(1\{\pi_i(0) \leq \mathbb{E}[\theta]\} - 1\{\pi_i(1) \leq \mathbb{E}[\theta]\}\right),\end{aligned}$$

which is independent of θ_{ik}. Since $\lim_{\theta\to\infty}\Delta(\theta) = \infty$, $\lim_{\theta\to-\infty}\Delta(\theta) = -\infty$, and $\frac{\partial\Delta(\theta)}{\partial\theta} > 0$, there exists a unique θ_i^* such that the director votes for the proposal if and only if $\theta_{ik} > \theta_i^*$. Thus, all directors within the firm follow the same threshold strategy.

It follows that $\tau_j^* = \tau(\theta_j^*)$, where $\tau(\theta) = \sum_{t=T}^{K} C_t^K \left(1 - F(\theta_i^*)\right)^t F(\theta_i^*)^{K-t}$, and $C_t^K = \frac{K!}{t!(K-t)!}$ is the Binomial coefficient. Note that $\tau(\theta)$ decreases with θ. Indeed, let $B_{K,p}(x)$ be the cumulative density function of a Binomial distribution with parameters (K,p). Then, by the properties of the Binomial distribution, $B_{K,p}(x)$ is first-order stochastically increasing as p increases, i.e., $B_{K,p_2}(x) < B_{K,p_1}(x)$ for $p_2 > p_1$. Note that $\tau(\theta) = 1 - B_{K,1-F(\theta)}(T-1)$. Since $F(\theta)$ increases with θ, $\tau(\theta)$ decreases with θ.

In the Online Appendix, we analyze the case where some directors are never pivotal. We show that no such equilibrium survives the trembling hand refinement, and the only trembling hand perfect equilibria are those where each director k in each firm i plays a threshold strategy and votes for the proposal if and only if $\theta_{ik} \geq \theta_i^*$ for some finite θ_i^*. ■

Proof of Lemma 2. Note that $\pi_i(\chi_i; \theta_i^*)$ is given by

$$\pi_i(\chi_i; \theta_i^*) = \begin{cases} \frac{\sum_{t=T}^{K} C_t^K (1-F(\theta_i^*))^t F(\theta_i^*)^{K-t} \left(\frac{t}{K}\mathbb{E}[\theta|\theta>\theta_i^*]+\frac{K-t}{K}\mathbb{E}[\theta|\theta\leq\theta_i^*]\right)}{\sum_{t=T}^{K} C_t^K (1-F(\theta_i^*))^t F(\theta_i^*)^{K-t}} & \text{if } \chi_i = 1 \\ \frac{\sum_{t=0}^{T-1} C_t^K (1-F(\theta_i^*))^t F(\theta_i^*)^{K-t} \left(\frac{t}{K}\mathbb{E}[\theta|\theta>\theta_i^*]+\frac{K-t}{K}\mathbb{E}[\theta|\theta\leq\theta_i^*]\right)}{\sum_{t=0}^{T-1} C_t^K (1-F(\theta_i^*))^t F(\theta_i^*)^{K-t}} & \text{if } \chi_i = 0 \end{cases} \tag{12}$$

Hence, $\pi_i(1; \theta_i^*)$ and $\pi_i(0; \theta_i^*)$ satisfy

$$\tau(\theta_i^*)\, \pi_i(1; \theta_i^*) + (1 - \tau(\theta_i^*))\, \pi_i(0; \theta_i^*) = \mathbb{E}[\theta], \tag{13}$$

where $\tau(\theta^*)$ is given by (5). Since $\pi_i(1; \theta_i^*)$ and $\pi_i(0; \theta_i^*)$ are weighted averages of terms where the smallest term in $\pi_i(1; \theta_i^*)$ is strictly higher than the largest term in $\pi_i(0; \theta_i^*)$, (6) must hold. ■

Proof of Lemma 3. A symmetric equilibrium exists if the equation $\beta(\theta^*) = \theta^*$ has a solution. Since $\beta(\cdot)$ is bounded and continuous, by the intermediate value theorem, a solution (not necessarily unique) always exists. To prove that all equilibria are symmetric, recall from the proof of Lemma 1 that all directors within a given firm i follow the same strategy with some threshold θ_i^*. Hence, it only remains to prove the symmetry of strategies across firms, i.e., that $\theta_i^* = \theta_j^*$. Suppose that there exists some asymmetric equilibrium in which $\theta_i^* > \theta_j^*$. In equilibrium, $\theta_i^* = \beta(\theta_j^*)$ and $\theta_j^* = \beta(\theta_i^*)$. Therefore, $\beta(\theta_j^*) > \beta(\theta_i^*)$. Since β is strictly increasing, this inequality implies $\theta_j^* > \theta_i^*$, which is a contradiction. ■

Proof of Proposition 1. Let $\beta(\theta, \alpha)$ denote the best response function for a given value of parameter α. Since $\tau(\theta)$ decreases in θ, $\lim_{\theta\to-\infty} \tau(\theta) = 1$ and $\lim_{\theta\to+\infty} \tau(\theta) = 0$, then for any given T, K there exists μ such that $\tau(\mu) = 0.5$. Then $\beta(\mu, \alpha) = \Delta^{-1}(0)$ for any

α, and $\tau(\theta^*) > 0.5 \Leftrightarrow \theta^* < \mu$. We prove parts (i) and (ii) here and relegate the proof of Part (iii) to the Online Appendix. Consider part (i). To prove that both a management-friendly and a shareholder-friendly equilibrium exist for a given α, we need to prove that the function $\Psi(\theta, \alpha) \equiv \beta(\theta, \alpha) - \theta$ has at least one root on $(\mu, +\infty)$ and at least one root on $(-\infty, \mu)$, where $\beta(\theta, \alpha)$ is given by(7). Since $\beta(\theta, \alpha)$ is bounded on $\theta \in (-\infty, +\infty)$, then $\lim_{\theta \to -\infty} \Psi(\theta, \alpha) = +\infty$ and $\lim_{\theta \to +\infty} \Psi(\theta, \alpha) = -\infty$. Hence, by the intermediate value theorem, both types of equilibria exist if there exist $\theta_1 < \mu$ and $\theta_2 > \mu$ such that $\Psi(\theta_1, \alpha) < 0 < \Psi(\theta_2, \alpha)$. We next show that this condition is satisfied for a large enough α. Fix any $\theta_1 < \mu$ and $\theta_2 > \mu$. By (7), $\Psi(\theta_1, \alpha)$ decreases in α and $\Psi(\theta_2, \alpha)$ increases in α. Moreover, $\lim_{\alpha \to \infty} \Psi(\theta_1, \alpha) = -\infty$ and $\lim_{\alpha \to \infty} \Psi(\theta_2, \alpha) = +\infty$. Hence, there exists $\hat{\alpha}$ such that for any $\alpha \geq \hat{\alpha}$, $\Psi(\theta_1, \alpha) < 0 < \Psi(\theta_2, \alpha)$. Hence, for any $\alpha \geq \hat{\alpha}$ there exists at least one shareholder-friendly and at least one management-friendly equilibrium. Consider the set $A = \{\hat{\alpha} \geq 0 :$ for any $\alpha \geq \hat{\alpha}$, there exists at least one shareholder-friendly and at least one management-friendly equilibrium$\}$. The above arguments prove that this set is non-empty. Then $\bar{\alpha}$ in the statement of the proposition is defined as $\inf\{A\}$.

Consider part (ii). First, we prove that for any $\alpha_0 < \bar{\alpha}$, all equilibria must be of the same type. Suppose, on the contrary, that both types of equilibria exist for some $\alpha_0 < \bar{\alpha}$, i.e., there exist $\theta_1 < \mu$ and $\theta_2 > \mu$ such that $\Psi(\theta_i, \alpha_0) = 0$. We show that in this case, both types of equilibria exist for any $\alpha > \alpha_0$ as well, which contradicts the definition of $\bar{\alpha}$ as $\inf\{A\}$. Indeed, for any $\alpha > \alpha_0$, $\Psi(\theta_1, \alpha) < \Psi(\theta_1, \alpha_0) = 0$ and $\Psi(\theta_2, \alpha) > \Psi(\theta_2, \alpha_0) = 0$. Since $\lim_{\theta \to -\infty} \Psi(\theta, \alpha) = +\infty$ and $\lim_{\theta \to +\infty} \Psi(\theta, \alpha) = -\infty$, then by the intermediate value theorem, there exist $\theta_1' \in (-\infty, \theta_1)$ and $\theta_2' \in (\theta_2, +\infty)$ such that $\Psi(\theta_i', \alpha) = 0$. These are the shareholder-friendly and management-friendly equilibria for $\alpha > \alpha_0$. Next, suppose $\Delta^{-1}(0) - \mu > (<)\, 0$. Consider any $\alpha_0 < \bar{\alpha}$. Since $\Psi(\mu, \alpha_0) = \Delta^{-1}(0) - \mu > (<)\, 0$ and $\lim_{\theta \to +\infty} \Psi(\theta, \alpha_0) = -\infty$ ($\lim_{\theta \to -\infty} \Psi(\theta, \alpha_0) = +\infty$), there exists $\hat{\theta} > (<)\, \mu$ such that $\Psi\left(\hat{\theta}, \alpha_0\right) = 0$, i.e., there exists at least one management-friendly (shareholder-friendly) equilibrium. Since, as shown above, all equilibria are of the same type, then all equilibria must be management-friendly (shareholder-friendly). ∎

Proof of Proposition 2. Consider any parameter p. Let $\beta(\theta, p)$ denote the best response function for a given value of this parameter, and $\bar{\theta}^*(p)$ and $\underline{\theta}^*(p)$ denote the greatest and the least fixed points of $\beta(\theta, p)$. In the Online Appendix, we show that $\bar{\theta}^*(p)$ and $\underline{\theta}^*(p)$ exist, and that $\bar{\theta}^*(p) = \sup \bar{S}(p)$, where $\bar{S}(p) = \{\theta : \beta(\theta, p) \geq \theta\}$, and $\underline{\theta}^*(p) = \inf \underline{S}(p)$, where $\underline{S}(p) = \{\theta : \beta(\theta, p) \leq \theta\}$.

Consider part (i). In the Online Appendix, we show that if $\theta^* \in \{\bar{\theta}^*, \underline{\theta}^*\}$, then $\frac{\partial \beta(\theta,\alpha)}{\partial \theta}|_{\theta=\theta^*} < 1$. Using the implicit function theorem for the equality $\beta(\theta^*, \alpha) - \theta^* = 0$, we get $\frac{\partial \theta^*}{\partial \alpha} = \frac{\frac{\partial \beta(\theta,\alpha)}{\partial \alpha}|_{\theta=\theta^*}}{1 - \frac{\partial \beta(\theta,\alpha)}{\partial \theta}|_{\theta=\theta^*}}$. Since $\frac{\partial \beta(\theta,\alpha)}{\partial \theta}|_{\theta=\theta^*} < 1$, then $sgn\left(\frac{\partial \theta^*}{\partial \alpha}\right) = sgn\left(\frac{\partial \beta(\theta,\alpha)}{\partial \alpha}|_{\theta=\theta^*}\right)$. Therefore, since $\Delta^{-1}(\cdot)$ is a strictly increasing function, $\frac{\partial \theta^*}{\partial \alpha} > 0 \Leftrightarrow \frac{\partial \beta(\theta,\alpha)}{\partial \alpha} > 0 \Leftrightarrow \tau(\theta^*) < 0.5$. Since $\tau(\cdot)$ is a decreasing function, then $\frac{\partial \tau(\theta^*)}{\partial \alpha} > 0 \Leftrightarrow \frac{\partial \theta^*}{\partial \alpha} < 0 \Leftrightarrow \tau(\theta^*) > 0.5$.

Next, in the Online Appendix, we also show that for any parameter p, if $\beta(\theta, p)$ increases with p, then $\bar{\theta}^*(p)$ and $\underline{\theta}^*(p)$ increase with p as well. We will now use this result to prove statements (ii), (iii), (iv.a), and (iv.b). Consider part (ii). Let $B_{K,p}(x)$ be the cumulative density function of a Binomial distribution with parameters (K, p). Then, by the properties of the Binomial distribution, $B_{K,p}(x)$ is first-order stochastically increasing as p increases, i.e., $B_{K,p_2}(x) < B_{K,p_1}(x)$ for $p_2 > p_1$. Let parameter κ parameterize distribution F, such that $F(\kappa_2, \theta)$ first-order stochastically dominates $F(\kappa_1, \theta)$ for $\kappa_2 > \kappa_1$. Then, for any θ, as κ increases, $1 - F(\theta)$ increases and $B_{K,1-F(\theta)}(\cdot)$ decreases. Since $\tau(\theta) = 1 - B_{K,1-F(\theta)}(T-1)$, then $\tau(\theta)$ increases with κ for any θ, and hence $\beta(\theta, \kappa)$ decreases with κ. By the result above, this implies that θ^* decreases with κ as well. Thus, $\tau(\theta^*)$ increases with κ for two reasons: first, because θ^* decreases with κ and second, because $\tau(\theta)$ increases with κ for any given θ. The implicit assumption in this analysis is that the distribution of outside candidates changes with κ as well, so that the expectations of the two distributions remain equal. Consider part (iii). Since $\tau(\theta) = 1 - B_{K,1-F(\theta)}(T-1)$, the function $\tau(\cdot)$ decreases with T by the properties of the Binomial distribution. Since $\beta(\theta) = \Delta^{-1}\left(\frac{\alpha\delta}{K}(1 - 2\tau(\theta))\right)$, the function $\beta(\cdot)$ increases with T. Thus, by the result above, θ^* increases with T, and hence $\tau(\theta^*)$ decreases with T, both because $1 - F(\theta^*)$ decreases and because $\tau(\theta)$ decreases with T for any θ. Consider part (iv.a). Note that $B_{K_2,p}(x) < B_{K_1,p}(x)$ for $K_2 > K_1$. Hence, $\tau(\cdot)$ increases with K, and thus $\beta(\cdot) = \Delta^{-1}(\alpha\lambda(1 - 2\tau(\cdot)))$ decreases with K for a fixed T. By the result above, this implies that θ^* decreases with K. Hence, $\tau(\theta^*) = 1 - B_{K,1-F(\theta^*)}(T-1)$ increases with K, both because $1 - F(\theta^*)$ increases and because $\tau(\theta)$ increases with K for any θ. Consider part (iv.b). If $T = K$, then $\tau(\theta) = (1 - F(\theta))^K$, which decreases with K for any θ. Hence, $\beta(\theta)$ increases with K. By the result above, this implies that θ^* increases with K. Hence, $\tau(\theta^*) = (1 - F(\theta^*))^K$decreases with K, both because $1 - F(\theta^*)$ decreases and because $(1 - F(\theta))^K$ decreases with K for any θ.

Finally, consider part (iv.c). Consider K_1 and K_2, $K_1 < K_2$, and suppose that $\alpha > \bar{\alpha}(K_1)$, where $\bar{\alpha}(K_1)$ is given by Proposition 1 when $K = K_1$. Since $\alpha > \bar{\alpha}(K_1)$, both types of equilibria co-exist, and hence the equilibrium characterized by $\underline{\theta}^*(K_1)$ is shareholder-friendly, which implies $\tau(\underline{\theta}^*(K_1)) > 0.5$ and $F(\underline{\theta}^*(K_1)) < 0.5$, and the equilibrium characterized by $\bar{\theta}^*(K_1)$ is management-friendly, which implies $\tau\left(\bar{\theta}^*(K_1)\right) < 0.5$ and $F\left(\bar{\theta}^*(K_1)\right) > 0.5$. First, consider $\underline{\theta}^*(K)$. Note that $\tau(\theta, K) = g(1 - F(\theta), K)$, where $g(p, K) = \sum_{t=\frac{K+1}{2}}^{K} C_t^K p^t (1-p)^{K-t}$. Since $1 - F(\underline{\theta}^*(K_1)) > 0.5$, then, according to the supplementary Lemma 7 in the Online Appendix, $g(1 - F(\underline{\theta}^*(K_1)), K_2) > g(1 - F(\underline{\theta}^*(K_1)), K_1)$, and hence $\tau(\underline{\theta}^*(K_1), K_2) > \tau(\underline{\theta}^*(K_1), K_1)$. Since $\beta(\theta, K) = \Delta^{-1}(\alpha\lambda(1 - 2\tau(\theta, K)))$, then $\beta(\underline{\theta}^*(K_1), K_2) < \beta(\underline{\theta}^*(K_1), K_1)$. Therefore, $\beta(\underline{\theta}^*(K_1), K_2) - \underline{\theta}^*(K_1) < \beta(\underline{\theta}^*(K_1), K_1) - \underline{\theta}^*(K_1) = 0$. Hence, $\underline{\theta}^*(K_1) \in \underline{S}(K_2)$, and since $\underline{\theta}^*(K_2) = \inf \underline{S}(K_2)$, then $\underline{\theta}^*(K_2) \leq \underline{\theta}^*(K_1)$. Thus, indeed, the most shareholder-friendly equilibrium becomes even more shareholder-friendly as K increases. Next, consider $\bar{\theta}^*(K)$. Since $1 - F\left(\bar{\theta}^*(K_1)\right) < 0.5$, then, according to Lemma 7, $g(1 - F\left(\bar{\theta}^*(K_1)\right), K_2) < g(1 - F\left(\bar{\theta}^*(K_1)\right), K_1)$. Hence, $\tau\left(\bar{\theta}^*(K_1), K_2\right) < \tau\left(\bar{\theta}^*(K_1), K_1\right)|$, and thus $\beta\left(\bar{\theta}^*(K_1), K_2\right) > \beta\left(\bar{\theta}^*(K_1), K_1\right)$. Therefore, $\beta\left(\bar{\theta}^*(K_1), K_2\right) - \bar{\theta}^*(K_1) > \beta\left(\bar{\theta}^*(K_1), K_1\right) - \bar{\theta}^*(K_1) = 0$. Hence, $\bar{\theta}^*(K_1) \in \bar{S}(K_2)$, and since $\bar{\theta}^*(K_2) = \sup \bar{S}(K_2)$, then $\bar{\theta}^*(K_1) \leq \bar{\theta}^*(K_2)$. Thus, indeed, the least shareholder-friendly equilibrium becomes even less shareholder-friendly as K increases. ■

Proof of Lemma 4. The first statement of the lemma follows from the proof of Proposition 2 (iv.c). That proof shows that if $\bar{\theta}^*(K)$ is management-friendly and $\underline{\theta}^*(K)$ is shareholder-friendly (which is always the case if both types of equilibria co-exist), then $\bar{\theta}^*(K)$ increases and $\underline{\theta}^*(K)$ decreases as K increases, and hence both types of equilibria continue to exist.

Consider the second statement, i.e., suppose that $\Delta(\mathbb{E}[\theta]) \neq 0$. Define

$$\begin{aligned}\alpha_1 &\equiv \frac{1}{\lambda}\max\{-\Delta(\mu), \Delta(\mu)\}\\ \alpha_2 &\equiv \bar{\alpha}(3),\end{aligned}$$

where $\bar{\alpha}(K) < \infty$ is $\bar{\alpha}$ defined in Proposition 1 when board size is K, and μ satisfies $\tau(\mu, K) = \frac{1}{2}$. An equilibrium is shareholder-friendly if and only if $\theta^* < \mu$. Note that under a simple majority rule, $\tau(\theta, K) > \frac{1}{2} \Leftrightarrow F(\theta^*) < \frac{1}{2}$, and hence $\mu = F^{-1}\left(\frac{1}{2}\right)$ for all K. In particular, for a symmetric distribution, $\mu = \mathbb{E}[\theta]$, and hence $\alpha_1 > 0$.

Consider part (iii). In the Online Appendix, we prove that if $\Delta(\mu) \neq 0$, then $\bar{\alpha} \in A$, where $A = \{\hat{\alpha} \geq 0 :$ for any $\alpha \geq \hat{\alpha}$, there exists at least one shareholder-friendly and at least one management-friendly equilibrium$\}$. Thus, if $\Delta(\mu) \neq 0$, both types of equilibria exist for $\alpha \geq \bar{\alpha}(3)$ when $K = 3$. Hence, based on the first statement of the lemma, both types of equilibria exist for $\alpha \geq \bar{\alpha}(3)$ and any $K > 3$, as required. Consider part (i). Since $\tau(\theta, K) \in (0, 1)$, then $\beta(\theta, K) \in (\Delta^{-1}(-\alpha\lambda), \Delta^{-1}(\alpha\lambda))$ for any θ and $K \geq 3$. Since any equilibrium θ^* is a solution to $\beta(\theta^*, K) = \theta^*$, then any equilibrium satisfies $\theta^* \in (\Delta^{-1}(-\alpha\lambda), \Delta^{-1}(\alpha\lambda))$. Note that $\alpha \leq \alpha_1 \Leftrightarrow \mu \notin (\Delta^{-1}(-\alpha\lambda), \Delta^{-1}(\alpha\lambda))$. If $\mu \leq \Delta^{-1}(-\alpha\lambda)$, then any equilibrium satisfies $\theta^* > \mu$, i.e., is management-friendly. Similarly, if $\mu \geq \Delta^{-1}(\alpha\lambda)$, then any equilibrium satisfies $\theta^* < \mu$, i.e., is shareholder-friendly, which completes the proof of part (i). Note also that parts (i) and (iii) together imply that $\alpha_1 < \alpha_2$. Finally, part (ii) is proved in the Online Appendix. ■

Proof of Lemma 5. Using (13), we can rewrite the expression for $c(\theta^*)$ as $c(\theta^*) = \mathbb{E}[\theta] + \delta\xi(\theta^*)$, where

$$\xi(\theta^*) = (2\tau(\theta^*) - 1)\,\tau(\theta^*)\,\frac{\pi(1; \theta^*) - \mathbb{E}[\theta]}{K}. \tag{14}$$

Consider Part (i). If $\tau(\theta^*) = \frac{1}{2}$, then it follows directly from the expression of $\xi(\theta^*)$ that

$\xi(\theta^*) = 0$. The case $\tau(\theta^*) = 1$ requires $\theta^* \to -\infty$. Note that

$$
\begin{aligned}
&\lim_{\theta^*\to-\infty} \tau(\theta^*)\left(\pi(1;\theta^*) - \mathbb{E}[\theta]\right) \\
&= \lim_{\theta^*\to-\infty} \textstyle\sum_{t=T}^{K} C_t^K \left(1 - F(\theta^*)\right)^t F(\theta^*)^{K-t} \left(\frac{t}{K}\mathbb{E}[\theta|\theta > \theta^*] + \frac{K-t}{K}\mathbb{E}[\theta|\theta < \theta^*] - \mathbb{E}[\theta]\right) \\
&= \lim_{\theta^*\to-\infty} \textstyle\sum_{t=T}^{K} C_t^K \left(1 - F(\theta^*)\right)^t F(\theta^*)^{K-t}\ \frac{t}{K}\mathbb{E}[\theta|\theta > \theta^*] \\
&- \lim_{\theta^*\to-\infty} \textstyle\sum_{t=T}^{K} C_t^K \left(1 - F(\theta^*)\right)^t F(\theta^*)^{K-t}\, \mathbb{E}[\theta] \\
&+ \lim_{\theta^*\to-\infty} \textstyle\sum_{t=T}^{K} C_t^K \left(1 - F(\theta^*)\right)^t F(\theta^*)^{K-t}\ \frac{K-t}{K}\mathbb{E}[\theta|\theta < \theta^*]
\end{aligned}
$$

Since $\lim_{\theta^*\to-\infty} F(\theta^*) = 0$ and $\lim_{\theta^*\to-\infty} \mathbb{E}[\theta|\theta > \theta^*] = \mathbb{E}[\theta]$, both the first and second terms equal $\mathbb{E}[\theta]$ and hence cancel out. The third term can be rewritten as

$$
\begin{aligned}
&\lim_{\theta^*\to-\infty} \left(F(\theta^*)\,\mathbb{E}[\theta|\theta < \theta^*]\right) \times \lim_{\theta^*\to-\infty} \frac{1}{F(\theta^*)} \sum_{t=T}^{K} C_t^K F(\theta^*)^{K-t}\, \frac{K-t}{K} \\
= \quad &\lim_{\theta^*\to-\infty} \int_{-\infty}^{\theta^*} x dF(x) \times \lim_{\theta^*\to-\infty} \sum_{t=T}^{K-1} C_t^K F(\theta^*)^{K-1-t}\, \frac{K-t}{K} = 0 \times 1 = 0.
\end{aligned}
$$

The case $\tau(\theta^*) = 0$ requires $\theta^* \to \infty$. Similarly to above,

$$
\begin{aligned}
&\lim_{\theta^*\to\infty} \tau(\theta^*)\left(\pi(1;\theta^*) - \mathbb{E}[\theta]\right) \\
&= \lim_{\theta^*\to\infty} \textstyle\sum_{t=T}^{K} C_t^K \left(1 - F(\theta^*)\right)^t F(\theta^*)^{K-t} \left(\frac{t}{K}\mathbb{E}[\theta|\theta > \theta^*] + \frac{K-t}{K}\mathbb{E}[\theta|\theta < \theta^*] - \mathbb{E}[\theta]\right) \\
&= \lim_{\theta^*\to\infty} \textstyle\sum_{t=T}^{K} C_t^K \left(1 - F(\theta^*)\right)^t F(\theta^*)^{K-t}\ \frac{t}{K}\mathbb{E}[\theta|\theta > \theta^*] \\
&= \lim_{\theta^*\to\infty} \left(\left(1 - F(\theta^*)\right)\mathbb{E}[\theta|\theta > \theta^*]\right) \times \lim_{\theta^*\to\infty} \frac{1}{1-F(\theta^*)} \textstyle\sum_{t=T}^{K} C_t^K \left(1 - F(\theta^*)\right)^t \frac{t}{K} \\
&= \lim_{\theta^*\to\infty} \textstyle\int_{\theta^*}^{\infty} x dF(x) \times \lim_{\theta^*\to\infty} \sum_{t=T}^{K} C_t^K \left(1 - F(\theta^*)\right)^{t-1} \frac{t}{K} = 0 \times 0 = 0.
\end{aligned}
$$

This concludes part (i). Parts (ii) and (iii) follow from the expression for $\xi(\theta^*)$ and the observation that $\pi(1;\theta^*) > \mathbb{E}[\theta]$ for any $\theta^* > -\infty$. ∎

Proof of Lemma 6. The first statement follows from Lemma 5, which implies that shareholders' (management's) expected utility in any shareholder-friendly equilibrium is strictly higher (lower) than in any management-friendly equilibrium. To prove the second statement, recall that $g_{SH}(\cdot)$ and $g_M(\cdot)$ are linear functions. Then, for any θ_1^* and θ_2^*,

$$
\begin{aligned}
& W_{Shareholders}\left(\theta_2^*\right) > W_{Shareholders}\left(\theta_1^*\right) \Leftrightarrow \tau\left(\theta_2^*\right) \times v_{SH}\left(1\right) + g'_{SH} \cdot c\left(\theta_2^*\right) > \tau\left(\theta_1^*\right) \times v_{SH}\left(1\right) + g'_{SH} \cdot c\left(\theta_1^*\right) \\
& \Leftrightarrow \left(\tau\left(\theta_2^*\right) - \tau\left(\theta_1^*\right)\right) \tfrac{v_{SH}(1)}{g'_{SH}} > \left(c\left(\theta_1^*\right) - c\left(\theta_2^*\right)\right) \Leftrightarrow \left(\tau\left(\theta_2^*\right) - \tau\left(\theta_1^*\right)\right) \tfrac{v_M(0)}{-g'_M} > \left(c\left(\theta_1^*\right) - c\left(\theta_2^*\right)\right) \\
& \Leftrightarrow \left(1 - \tau\left(\theta_2^*\right)\right) \times v_M\left(0\right) + g'_M \cdot c\left(\theta_2^*\right) < \left(1 - \tau\left(\theta_1^*\right)\right) \times v_M\left(0\right) + g'_M \cdot c\left(\theta_1^*\right) \\
& \Leftrightarrow W_{Management}\left(\theta_2^*\right) < W_{Management}\left(\theta_1^*\right),
\end{aligned}
$$

which proves Pareto efficiency. ■